Embedding Bounded Bandwidth Graphs into ℓ_p UCLA Tech Report

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Abstract

We prove that graphs can be embedded randomly into ℓ_p (for any $p \geq 1$) with expected distortion polynomial in k, where k is the *bandwidth* of the graph. This implies the existence of an infinite family of graphs which embed well into ℓ_p, which is surprising since *trees* do not form such a class, having a lower bound of $\Omega(\log \log n)$ for ℓ_2 embedding. Our results extend to graphs of bounded *tree-bandwidth* when the target metric is ℓ_1, or when we allow an additional term of size $O(\log \log n)$ in our distortion.

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