Featherweight eJava

Alessandro Warth and Todd Millstein
Computer Science Department
University of California, Los Angeles
{awarth,todd}@cs.ucla.edu

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1 Introduction

This paper details Featherweight eJava (FeJ), an extension of Featherweight Java (FJ) that formally models the essential aspects of eJava. eJava is an extension of Java containing *expanders*, a new language construct that supports object adaptation. Expanders and the eJava language, as well as an introduction to the FeJ formalism, are reported on in a companion OOPSLA 2006 paper [1].

2 Conventions

The metavariables `S`, `T`, and `U` range over all type names, including classes, interfaces, and expanders; `C` and `D` range over class names; `I` and `J` range over interface names; `X` and `Y` range over expander names; `f` and `g` range over field names; `m` ranges over method names; `x` ranges over parameter names; `s` and `t` range over terms; `u` and `v` range over values; `TD` ranges over type declarations, including class, interface, and expander declarations; `K` ranges over constructor declarations; `M` ranges over method declarations; `MH` ranges over method headers.

We share the sanity conditions of FJ and generalize them to our context in the natural way. We also require all types expanded by a given expander `X` (either as the "top" type or in an overriding expander) to be distinct.

3 Syntax

$$\begin{array}{lll}\texttt{TD} & ::= & \texttt{class C extends D implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{; K } \overline{\texttt{M}}\} \\ & | & \texttt{interface I extends } \overline{\texttt{I}}\ \{\overline{\texttt{MH}}\} \\ & | & \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{; } \overline{\texttt{M}}\}\ \overline{\texttt{O}}\end{array}$$

$$\texttt{O} ::= \texttt{of C } \{\overline{\texttt{M}}\}$$

```
K ::= C(T̄ f̄) {super(f̄); this.f̄=f̄;}

M ::= T m(T̄ x̄) {return t;}

MH ::= T m(T̄ x̄);

T ::= C | I | T^X

t ::= x
    | t.f
    | t.m(f̄)
    | new C(t̄)
    | (T) t
    | t with X
    | peel t1

v ::= new C(v̄)
    | v with X
```

4 Subtyping

$\boxed{\mathtt{S} \lhd \mathtt{T}}$

$$\mathtt{T} \lhd \mathtt{T} \qquad \text{(S-REF)}$$

$$\frac{\mathtt{S} \lhd \mathtt{T} \qquad \mathtt{T} \lhd \mathtt{U}}{\mathtt{S} \lhd \mathtt{U}} \qquad \text{(S-TRANS)}$$

$$\frac{\mathtt{S} \lhd \mathtt{T}}{\mathtt{S}^{\mathtt{X}} \lhd \mathtt{T}^{\mathtt{X}}} \qquad \text{(S-EXPAND)}$$

$$\frac{TT(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements\ } \overline{\mathtt{I}}\ \{\ldots\}}{\mathtt{C} \lhd \mathtt{D}} \qquad \text{(S-CLS1)}$$

$$\frac{TT(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements\ } \overline{\mathtt{I}}\ \{\ldots\}}{\mathtt{C} \lhd \mathtt{I}_i} \qquad \text{(S-CLS2)}$$

$$\frac{TT(\mathtt{I}) = \mathtt{interface\ I\ extends\ } \overline{\mathtt{J}}\ \{\ldots\}}{\mathtt{I} \lhd \mathtt{J}_i} \qquad \text{(S-INT)}$$

$$\frac{TT(\mathtt{X}) = \mathtt{expander\ X\ of\ T\ implements\ } \overline{\mathtt{I}}\ \{\ldots\}\ \overline{\mathtt{O}}}{\mathtt{T}^{\mathtt{X}} \lhd \mathtt{I}_i} \qquad \text{(S-EXP)}$$

5 Dynamic Semantics

5.1 Field lookup

$\boxed{fields(\texttt{C}) = \overline{\texttt{T}}\ \overline{\texttt{f}}}$

$$fields(\texttt{Object}) = \bullet$$

$$\frac{TT(\texttt{C}) = \texttt{class C extends D implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{; K } \overline{\texttt{M}}\} \qquad fields(\overline{\texttt{D}}) = \overline{\texttt{U}}\ \overline{\texttt{g}}}{fields(\texttt{C}) = \overline{\texttt{U}}\ \overline{\texttt{g}}, \overline{\texttt{T}}\ \overline{\texttt{f}}} \tag{FieldsC}$$

$\boxed{fields(\texttt{X}) = \overline{\texttt{T}}\ \overline{\texttt{f}}\ \texttt{=}\ \overline{\texttt{v}}}$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\ \texttt{=}\ \overline{\texttt{v}}\texttt{; } \overline{\texttt{M}}\}\ \overline{\texttt{O}}}{fields(\texttt{X}) = \overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}} \tag{FieldsX}$$

5.2 Method Body Lookup

$\boxed{mbody(\texttt{m}, \texttt{C}) = (\overline{\texttt{x}}, \texttt{t})}$

$$\frac{TT(\texttt{C}) = \texttt{class C extends D implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{; K } \overline{\texttt{M}}\} \qquad \texttt{U m(}\overline{\texttt{U}}\ \overline{\texttt{x}}\texttt{) \{return t;\}} \in \overline{\texttt{M}}}{mbody(\texttt{m}, \texttt{C}) = (\overline{\texttt{x}}, \texttt{t})} \tag{MBody-C1}$$

$$\frac{TT(\texttt{C}) = \texttt{class C extends D implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{; K } \overline{\texttt{M}}\} \qquad \texttt{m} \text{ is not defined in } \overline{\texttt{M}}}{mbody(\texttt{m}, \texttt{C}) = mbody(\texttt{m}, \texttt{D})} \tag{MBody-C2}$$

$\boxed{mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{D}) = (\overline{\texttt{x}}, \texttt{t})}$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{; } \overline{\texttt{M}}\}\ \overline{\texttt{O}} \qquad \texttt{of D } \{\overline{\texttt{M}'}\} \in \overline{\texttt{O}} \qquad \texttt{U m(}\overline{\texttt{U}}\ \overline{\texttt{x}}\texttt{) \{return t;\}} \in \overline{\texttt{M}'}}{mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{D}) = (\overline{\texttt{x}}, \texttt{t})} \tag{MBody-X1}$$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{; } \overline{\texttt{M}}\}\ \overline{\texttt{O}} \qquad \texttt{of D } \{\overline{\texttt{M}'}\} \in \overline{\texttt{O}} \qquad \texttt{m} \text{ is not defined in } \overline{\texttt{M}'} \qquad TT(\texttt{D}) = \texttt{class D extends E } \cdots}{mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{D}) = mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{E})} \tag{MBody-X2}$$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{; } \overline{\texttt{M}}\}\ \overline{\texttt{O}} \qquad \texttt{D} \text{ is not defined in } \overline{\texttt{O}} \qquad TT(\texttt{D}) = \texttt{class D extends E } \cdots}{mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{D}) = mbody(\texttt{m}, \texttt{X}, \texttt{C}, \texttt{E})} \tag{MBody-X3}$$

$$\frac{\begin{array}{c}\mathit{TT}(\mathtt{X}) = \mathtt{expander\ X\ of\ T\ implements}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{v}};\ \overline{\mathtt{M}}\}\ \overline{\mathtt{O}} \\ \mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}})\ \{\mathtt{return\ t;}\} \in \overline{\mathtt{M}}\end{array}}{\mathit{mbody}(\mathtt{m}, \mathtt{X}, \mathtt{C}, \mathtt{Object}) = (\overline{\mathtt{x}}, \mathtt{t})} \quad \text{(MBody-X4)}$$

5.3 Term Evaluation

$\boxed{\mathtt{t} \longrightarrow \mathtt{t}'}$

$$\frac{\mathit{fields}(\mathtt{C}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\mathtt{(new\ C(}\overline{\mathtt{v}}\mathtt{)).f}_i \longrightarrow \mathtt{v}_i} \quad \text{(E-ProjNew)}$$

$$\frac{\mathit{fields}(\mathtt{X}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{v}}}{\mathtt{(v\ with\ X).f}_i \longrightarrow \mathtt{v}_i} \quad \text{(E-ProjWith1)}$$

$$\frac{\mathit{fields}(\mathtt{X}) = \overline{\mathtt{T}}\ \overline{\mathtt{g}}\mathtt{=}\overline{\mathtt{v}} \qquad \mathtt{f} \notin \overline{\mathtt{g}}}{\mathtt{(v\ with\ X).f} \longrightarrow \mathtt{v.f}} \quad \text{(E-ProjWith2)}$$

$$\frac{\mathit{mbody}(\mathtt{m}, \mathtt{C}) = (\overline{\mathtt{x}}, \mathtt{t}_0)}{\mathtt{(new\ C(}\overline{\mathtt{v}}\mathtt{)).m(}\overline{\mathtt{u}}\mathtt{)} \longrightarrow [\overline{\mathtt{x}} \mapsto \overline{\mathtt{u}}, \mathtt{this} \mapsto \mathtt{new\ C(}\overline{\mathtt{v}}\mathtt{)}]\mathtt{t}_0} \quad \text{(E-InvkNew)}$$

$$\frac{\mathtt{v} = \mathtt{new\ C(}\overline{\mathtt{v}}\mathtt{)} \qquad \mathit{mbody}(\mathtt{m}, \mathtt{X}, \mathtt{C}, \mathtt{C}) = (\overline{\mathtt{x}}, \mathtt{t}_0)}{\mathtt{(v\ with\ X).m(}\overline{\mathtt{u}}\mathtt{)} \longrightarrow [\overline{\mathtt{x}} \mapsto \overline{\mathtt{u}}, \mathtt{this} \mapsto \mathtt{v\ with\ X}]\mathtt{t}_0} \quad \text{(E-InvkWith1)}$$

$$\frac{\mathtt{v} = \mathtt{v'\ with\ X'} \qquad \mathit{mbody}(\mathtt{m}, \mathtt{X}, \mathtt{Object}, \mathtt{Object}) = (\overline{\mathtt{x}}, \mathtt{t}_0)}{\mathtt{(v\ with\ X).m(}\overline{\mathtt{u}}\mathtt{)} \longrightarrow [\overline{\mathtt{x}} \mapsto \overline{\mathtt{u}}, \mathtt{this} \mapsto \mathtt{v\ with\ X}]\mathtt{t}_0} \quad \text{(E-InvkWith2)}$$

$$\frac{\begin{array}{c}\mathit{TT}(\mathtt{X}) = \mathtt{expander\ X\ of\ T\ implements}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{v}};\ \overline{\mathtt{M}}\}\ \overline{\mathtt{O}} \\ \mathtt{m} \text{ is not defined in } \overline{\mathtt{M}}\end{array}}{\mathtt{(v\ with\ X).m(}\overline{\mathtt{u}}\mathtt{)} \longrightarrow \mathtt{v.m(}\overline{\mathtt{u}}\mathtt{)}} \quad \text{(E-InvkWith3)}$$

$$\frac{\bullet \vdash \mathtt{v} : \mathtt{S} \qquad \mathtt{S} \lhd \mathtt{U}}{\mathtt{(U)(v)} \longrightarrow \mathtt{v}} \quad \text{(E-CastVal)}$$

$$\frac{\mathtt{t}_0 \longrightarrow \mathtt{t}_0'}{\mathtt{t}_0\mathtt{.f} \longrightarrow \mathtt{t}_0'\mathtt{.f}} \quad \text{(E-Field)}$$

$$\frac{\mathtt{t}_0 \longrightarrow \mathtt{t}_0'}{\mathtt{t}_0\mathtt{.m(}\overline{\mathtt{t}}\mathtt{)} \longrightarrow \mathtt{t}_0'\mathtt{.m(}\overline{\mathtt{t}}\mathtt{)}} \quad \text{(E-Invk-Recv)}$$

$$\frac{\mathtt{t}_i \longrightarrow \mathtt{t}_i'}{\mathtt{v}_0\mathtt{.m(}\overline{\mathtt{v}}, \mathtt{t}_i, \overline{\mathtt{t}}\mathtt{)} \longrightarrow \mathtt{v}_0\mathtt{.m(}\overline{\mathtt{v}}, \mathtt{t}_i', \overline{\mathtt{t}}\mathtt{)}} \quad \text{(E-Invk-Arg)}$$

$$\frac{\mathtt{t}_i \longrightarrow \mathtt{t}_i'}{\mathtt{new\ C(}\overline{\mathtt{v}}, \mathtt{t}_i, \overline{\mathtt{t}}\mathtt{)} \longrightarrow \mathtt{new\ C(}\overline{\mathtt{v}}, \mathtt{t}_i', \overline{\mathtt{t}}\mathtt{)}} \quad \text{(E-New-Arg)}$$

$$\frac{\mathtt{t}_0 \longrightarrow \mathtt{t}_0'}{(\mathtt{U})\mathtt{t}_0 \longrightarrow (\mathtt{U})\mathtt{t}_0'} \qquad \text{(E-Cast)}$$

$$\frac{\mathtt{t}_0 \longrightarrow \mathtt{t}_0'}{\mathtt{t}_0\ \mathtt{with}\ \mathtt{X} \longrightarrow \mathtt{t}_0'\ \mathtt{with}\ \mathtt{X}} \qquad \text{(E-With)}$$

$$\frac{\mathtt{t}_0 \longrightarrow \mathtt{t}_0'}{\mathtt{peel}\ \mathtt{t}_0 \longrightarrow \mathtt{peel}\ \mathtt{t}_0'} \qquad \text{(E-Peel)}$$

$$\mathtt{peel}\ (\mathtt{v}\ \mathtt{with}\ \mathtt{X}) \longrightarrow \mathtt{v} \qquad \text{(E-PeelWith)}$$

6 Static Semantics

6.1 Field Type Lookup

$\boxed{\mathit{ftype}(\mathtt{f}, \mathtt{T}) = \mathtt{U}}$

$$\frac{\mathit{fields}(\mathtt{C}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\mathit{ftype}(\mathtt{f}_i, \mathtt{C}) = \mathtt{T}_i} \qquad \text{(FType1)}$$

$$\frac{\mathit{fields}(\mathtt{X}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{v}}}{\mathit{ftype}(\mathtt{f}_i, \mathtt{U}^{\mathtt{X}}) = \mathtt{T}_i} \qquad \text{(FType2)}$$

$$\frac{\mathit{fields}(\mathtt{X}) = \overline{\mathtt{T}}\ \overline{\mathtt{g}}\mathtt{=}\overline{\mathtt{v}} \qquad \mathtt{f}_i \notin \overline{\mathtt{g}}}{\mathit{ftype}(\mathtt{f}_i, \mathtt{U}^{\mathtt{X}}) = \mathit{ftype}(\mathtt{f}_i, \mathtt{U})} \qquad \text{(FType3)}$$

6.2 Method Type Lookup

$\boxed{\mathit{mtype}(\mathtt{m}, \mathtt{T}) = \overline{\mathtt{T}} \rightarrow \mathtt{T}}$

$$\frac{\begin{array}{c} \mathit{TT}(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}\} \\ \mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}})\ \{\mathtt{return\ t;}\} \in \overline{\mathtt{M}} \end{array}}{\mathit{mtype}(\mathtt{m}, \mathtt{C}) = \overline{\mathtt{U}} \rightarrow \mathtt{U}} \qquad \text{(MType-C1)}$$

$$\frac{\begin{array}{c} \mathit{TT}(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}\} \\ \mathtt{m}\ \text{is not defined in}\ \overline{\mathtt{M}} \end{array}}{\mathit{mtype}(\mathtt{m}, \mathtt{C}) = \mathit{mtype}(\mathtt{m}, \mathtt{D})} \qquad \text{(MType-C2)}$$

$$\frac{\begin{array}{c} \mathit{TT}(\mathtt{I}) = \mathtt{interface\ I\ extends}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{MH}}\} \\ \mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}})\mathtt{;} \in \overline{\mathtt{MH}} \end{array}}{\mathit{mtype}(\mathtt{m}, \mathtt{I}) = \overline{\mathtt{U}} \rightarrow \mathtt{U}} \qquad \text{(MType-I1)}$$

$$\frac{TT(\texttt{I}) = \texttt{interface I extends } \overline{\texttt{I}}\ \{\overline{\texttt{MH}}\} \qquad \texttt{m} \text{ is not defined in } \overline{\texttt{MH}}}{mtype(\texttt{m}, \texttt{I}) = mtype(\texttt{m}, \texttt{I}_i)} \quad \text{(MType-I2)}$$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{;}\ \overline{\texttt{M}}\}\ \overline{\texttt{O}} \qquad \texttt{U m}(\overline{\texttt{U}}\ \overline{\texttt{x}})\ \{\texttt{return t;}\} \in \overline{\texttt{M}}}{mtype(\texttt{m}, \texttt{S}^{\texttt{X}}) = \overline{\texttt{U}} \rightarrow \texttt{U}} \quad \text{(MType-X1)}$$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}}\texttt{;}\ \overline{\texttt{M}}\}\ \overline{\texttt{O}} \qquad \texttt{m} \text{ is not defined in } \overline{\texttt{M}}}{mtype(\texttt{m}, \texttt{S}^{\texttt{X}}) = mtype(\texttt{m}, \texttt{S})} \quad \text{(MType-X2)}$$

6.3 Term Typing

$\boxed{\Gamma \vdash \texttt{t} : \texttt{T}}$

$$\frac{\texttt{x} : \texttt{T} \in \Gamma}{\Gamma \vdash \texttt{x} : \texttt{T}} \quad \text{(T-Var)}$$

$$\frac{\Gamma \vdash \texttt{t} : \texttt{T} \qquad ftype(\texttt{f}, \texttt{T}) = \texttt{U}}{\Gamma \vdash \texttt{t.f} : \texttt{U}} \quad \text{(T-Field)}$$

$$\frac{\Gamma \vdash \texttt{t}_0 : \texttt{T}_0 \qquad mtype(\texttt{m}, \texttt{T}_0) = \overline{\texttt{T}} \rightarrow \texttt{T} \qquad \Gamma \vdash \overline{\texttt{t}} : \overline{\texttt{S}} \qquad \overline{\texttt{S}} \lhd \overline{\texttt{T}}}{\Gamma \vdash \texttt{t}_0\texttt{.m}(\overline{\texttt{t}}) : \texttt{T}} \quad \text{(T-Invk)}$$

$$\frac{fields(\texttt{C}) = \overline{\texttt{S}}\ \overline{\texttt{f}} \qquad \Gamma \vdash \overline{\texttt{t}} : \overline{\texttt{T}} \qquad \overline{\texttt{T}} \lhd \overline{\texttt{S}}}{\Gamma \vdash \texttt{new C}(\overline{\texttt{t}}) : \texttt{C}} \quad \text{(T-New)}$$

$$\frac{\Gamma \vdash \texttt{t}_0 : \texttt{S} \qquad \texttt{S} \lhd \texttt{T}}{\Gamma \vdash (\texttt{T})\texttt{t}_0 : \texttt{T}} \quad \text{(T-UCast)}$$

$$\frac{\Gamma \vdash \texttt{t}_0 : \texttt{S} \qquad \texttt{T} \lhd \texttt{S} \qquad \texttt{T} \neq \texttt{S}}{\Gamma \vdash (\texttt{T})\texttt{t}_0 : \texttt{T}} \quad \text{(T-DCast)}$$

$$\frac{\Gamma \vdash \texttt{t}_0 : \texttt{S} \qquad \texttt{T} \not\lhd \texttt{S} \qquad \texttt{S} \not\lhd \texttt{T} \qquad \textit{stupid warning}}{\Gamma \vdash (\texttt{T})\texttt{t}_0 : \texttt{T}} \quad \text{(T-SCast)}$$

$$\frac{TT(\texttt{X}) = \texttt{expander X of T implements } \overline{\texttt{I}}\ \{\ldots\}\ \overline{\texttt{O}} \qquad \Gamma \vdash \texttt{t} : \texttt{U} \qquad \texttt{U} \lhd \texttt{T}}{\Gamma \vdash \texttt{t with X} : \texttt{U}^{\texttt{X}}} \quad \text{(T-With)}$$

$$\frac{\Gamma \vdash \mathtt{t} : \mathtt{T}^{\mathtt{X}}}{\Gamma \vdash \mathtt{peel\ t} : \mathtt{T}} \quad \text{(T-Peel)}$$

6.4 Valid Method Overriding

$\boxed{\mathit{override}(\mathtt{m}, \mathtt{U}, \overline{\mathtt{T}} \to \mathtt{T}_0)}$

$$\frac{\mathit{mtype}(\mathtt{m}, \mathtt{U}) = \overline{\mathtt{U}} \to \mathtt{U}_0 \text{ implies } \overline{\mathtt{T}} = \overline{\mathtt{U}} \text{ and } \mathtt{T}_0 = \mathtt{U}_0}{\mathit{override}(\mathtt{m}, \mathtt{U}, \overline{\mathtt{T}} \to \mathtt{T}_0)} \quad \text{(Over1)}$$

$\boxed{\mathit{override}(\mathtt{m}, \mathtt{X}, \overline{\mathtt{T}} \to \mathtt{T}_0)}$

$$\frac{\begin{array}{c} \mathit{TT}(\mathtt{X}) = \mathtt{expander\ X\ of\ T\ implements}\ \overline{\mathtt{I}}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{v}};\ \overline{\mathtt{M}}\}\ \overline{\mathtt{O}} \\ \mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}})\ \{\mathtt{return\ t;}\} \in \overline{\mathtt{M}} \end{array}}{\mathit{override}(\mathtt{m}, \mathtt{X}, \overline{\mathtt{U}} \to \mathtt{U})} \quad \text{(Over2)}$$

6.5 Method Typing

$\boxed{\mathtt{M}\ \text{OK in}\ \mathtt{C}}$

$$\frac{\begin{array}{c} \overline{\mathtt{x}} : \overline{\mathtt{T}}, \mathtt{this} : \mathtt{C} \vdash \mathtt{t}_0 : \mathtt{U}_0 \qquad \mathtt{U}_0 \lhd \mathtt{T}_0 \\ \mathit{TT}(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements}\ \overline{\mathtt{I}}\ \{\ldots\} \\ \mathit{override}(\mathtt{m}, \mathtt{D}, \overline{\mathtt{T}} \to \mathtt{T}_0) \end{array}}{\mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \text{OK in}\ \mathtt{C}} \quad \text{(MethodOK)}$$

$\boxed{\mathtt{M}\ \text{OK in}\ \mathtt{X}, \mathtt{T}}$

$$\frac{\overline{\mathtt{x}} : \overline{\mathtt{T}}, \mathtt{this} : \mathtt{T}^{\mathtt{X}} \vdash \mathtt{t}_0 : \mathtt{U}_0 \qquad \mathtt{U}_0 \lhd \mathtt{T}_0}{\mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \text{OK in}\ \mathtt{X}, \mathtt{T}} \quad \text{(ExpMethodOK)}$$

$\boxed{\mathtt{M}\ \text{OverrideOK in}\ \mathtt{X}, \mathtt{C}}$

$$\frac{\begin{array}{c} \mathit{override}(\mathtt{m}, \mathtt{X}, \overline{\mathtt{T}} \to \mathtt{T}_0) \\ \mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \text{OK in}\ \mathtt{X}, \mathtt{C} \end{array}}{\mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \text{OverrideOK in}\ \mathtt{X}, \mathtt{C}} \quad \text{(OverrideOK)}$$

6.6 Interface Conformance

$\boxed{\mathit{reallyImplements}(\mathtt{T}, \mathtt{I})}$

$$\frac{\begin{array}{c} \mathit{TT}(\mathtt{I}) = \mathtt{interface\ I\ extends}\ \overline{\mathtt{J}}\ \{\overline{\mathtt{MH}}\} \\ \mathtt{S\ m}(\overline{\mathtt{S}}\ \overline{\mathtt{x}})\mathtt{;} \in \overline{\mathtt{MH}} \text{ implies } \mathit{mtype}(\mathtt{m}, \mathtt{T}) = \overline{\mathtt{U}} \to \mathtt{U} \text{ and } \mathit{override}(\mathtt{m}, \mathtt{I}, \overline{\mathtt{U}} \to \mathtt{U}) \\ \mathit{reallyImplements}(\mathtt{T}, \overline{\mathtt{J}}) \end{array}}{\mathit{reallyImplements}(\mathtt{T}, \mathtt{I})}$$

(ReallyImp)

6.7 Expander Overriding Typing

$\boxed{\texttt{O}\ \text{OK in}\ \texttt{X}}$

$$\frac{\begin{array}{c} TT(\texttt{X}) = \texttt{expander X of T implements}\ \overline{\texttt{I}}\ \{\ldots\}\ \overline{\texttt{O}} \\ \texttt{C} \lhd \texttt{T} \qquad \overline{\texttt{M}}\ \text{OverrideOK in}\ \texttt{X},\texttt{C} \end{array}}{\texttt{of C}\ \{\overline{\texttt{M}}\}\text{OK in}\ \texttt{X}} \quad \text{(OOK)}$$

6.8 Class, Interface, and Expander Typing

$\boxed{\texttt{TD}\ \text{OK}}$

$$\frac{\begin{array}{c} \texttt{K} = \texttt{C}(\overline{\texttt{U}}\ \overline{\texttt{g}},\ \overline{\texttt{T}}\ \overline{\texttt{f}})\ \{\texttt{super}(\overline{\texttt{g}});\ \texttt{this}.\overline{\texttt{f}}\texttt{=}\overline{\texttt{f}}\} \\ \mathit{fields}(\texttt{D}) = \overline{\texttt{U}}\ \overline{\texttt{g}} \qquad \overline{\texttt{M}}\ \text{OK in}\ \texttt{C} \\ \mathit{reallyImplements}(\texttt{C},\overline{\texttt{I}}) \end{array}}{\texttt{class C extends D implements}\ \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}};\ \texttt{K}\ \overline{\texttt{M}}\}\ \text{OK}} \quad \text{(COK)}$$

$$\frac{\begin{array}{c} \bullet \vdash \overline{\texttt{v}} : \overline{\texttt{S}} \qquad \overline{\texttt{S}} \lhd \overline{\texttt{T}} \\ \overline{\texttt{M}}\ \text{OK in}\ \texttt{X},\texttt{T} \qquad \overline{\texttt{O}}\ \text{OK in}\ \texttt{X} \\ \mathit{reallyImplements}(\texttt{T}^{\texttt{X}},\overline{\texttt{I}}) \end{array}}{\texttt{expander X of T implements}\ \overline{\texttt{I}}\ \{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{=}\overline{\texttt{v}};\ \overline{\texttt{M}}\}\ \overline{\texttt{O}}\ \text{OK}} \quad \text{(XOK)}$$

$$\frac{\mathit{reallyImplements}(\texttt{I},\overline{\texttt{J}})}{\texttt{interface I extends}\ \overline{\texttt{J}}\ \{\overline{\texttt{MH}}\}\ \text{OK}} \quad \text{(IOK)}$$

7 Type Soundness

Analogous with FJ, we assume that `TD OK` holds for each type declaration `TD` in the range of TT.

7.1 Type Preservation

Lemma 7.1 If $\texttt{S} \lhd \texttt{T}^{\texttt{X}}$, then `S` has the form $\texttt{U}^{\texttt{X}}$.
Proof By induction on the depth of the derivation of $\texttt{S} \lhd \texttt{T}^{\texttt{X}}$. Case analysis of the last rule in the derivation.

- Case S-Ref: Then $\texttt{S} = \texttt{T}^{\texttt{X}}$ and the result follows.
- Case S-Trans: Then $\texttt{S} \lhd \texttt{T}_0$ and $\texttt{T}_0 \lhd \texttt{T}^{\texttt{X}}$. By induction $\texttt{T}_0$ has the form $\texttt{U}_0^{\texttt{X}}$, and by induction again `S` has the form $\texttt{U}^{\texttt{X}}$.
- Case S-Expand: Then `S` has the form $\texttt{U}^{\texttt{X}}$.
- Case S-Cls1: Then we are given that $\texttt{T}^{\texttt{X}}$ has the form `C`, which is a contradiction.

- Case S-CLS2: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.
- Case S-INT: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.
- Case S-EXP: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.

Lemma 7.2 If $\mathtt{T} \lhd \mathtt{C}$, then T is a class.
Proof By induction on the depth of the derivation of $\mathtt{T} \lhd \mathtt{C}$. Case analysis of the last rule in the derivation.

- Case S-REF: Then T = C and the result follows.
- Case S-TRANS: Then $\mathtt{T} \lhd \mathtt{T}_0$ and $\mathtt{T}_0 \lhd \mathtt{C}$. By induction $\mathtt{T}_0$ is a class E, and by induction again T is a class.
- Case S-EXPAND: Then C has the form $\mathtt{S}^{\mathtt{X}}$, which is a contradiction.
- Case S-CLS1: Then we are given that T is a class.
- Case S-CLS2: Then we are given that T is a class.
- Case S-INT: Then C is an interface, contradicting our initial assumption.
- Case S-EXP: Then C is an interface, contradicting our initial assumption.

Lemma 7.3 If $\mathtt{Object} \lhd \mathtt{T}$, then T = Object.
Proof By induction on the depth of the derivation of $\mathtt{Object} \lhd \mathtt{T}$. Case analysis of the last rule in the derivation.

- Case S-REF: Then T = Object.
- Case S-TRANS: Then $\mathtt{Object} \lhd \mathtt{T}_0$ and $\mathtt{T}_0 \lhd \mathtt{T}$. By induction $\mathtt{T}_0 = \mathtt{Object}$, and by induction again T = Object.
- Case S-EXPAND: Then Object has the form $\mathtt{S}^{\mathtt{X}}$, which is a contradiction.
- Case S-CLS1: Then $\mathtt{Object} \in dom(TT)$, which contradicts an assumption about FeJ programs.
- Case S-CLS2: Then $\mathtt{Object} \in dom(TT)$, which contradicts an assumption about FeJ programs.
- Case S-INT: Then $\mathtt{Object} \in dom(TT)$, which contradicts an assumption about FeJ programs.
- Case S-EXP: Then Object has the form $\mathtt{S}^{\mathtt{X}}$, which is a contradiction.

Lemma 7.4 If $\mathtt{S}^{\mathtt{X}} \lhd \mathtt{T}^{\mathtt{X}}$, then $\mathtt{S} \lhd \mathtt{T}$.
Proof By induction on the depth of the derivation of $\mathtt{S}^{\mathtt{X}} \lhd \mathtt{T}^{\mathtt{X}}$. Case analysis of the last rule in the derivation.

- Case S-Ref: Then $\mathtt{S}^{\mathtt{X}} = \mathtt{T}^{\mathtt{X}}$, so $\mathtt{S} = \mathtt{T}$ and the result follows by S-Ref.
- Case S-Trans: Then $\mathtt{S}^{\mathtt{X}} \lhd \mathtt{T}_0$ and $\mathtt{T}_0 \lhd \mathtt{T}^{\mathtt{X}}$. By Lemma 7.1 $\mathtt{T}_0$ has the form $\mathtt{T}_1^{\mathtt{X}}$. Therefore, by induction we have $\mathtt{T}_1 \lhd \mathtt{T}$, and by induction again we have $\mathtt{S} \lhd \mathtt{T}_1$. Then the result follows by S-Trans.
- Case S-Expand: Then $\mathtt{S} \lhd \mathtt{T}$.
- Case S-Cls1: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form C, which is a contradiction.
- Case S-Cls2: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.
- Case S-Int: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.
- Case S-Exp: Then we are given that $\mathtt{T}^{\mathtt{X}}$ has the form I, which is a contradiction.

Lemma 7.5 If $\mathtt{D} \lhd \mathtt{C}$ and *fields*(C) = $\overline{\mathtt{T}}\ \overline{\mathtt{f}}$, then *fields*(C) $\subseteq$ *fields*(D).
Proof By induction on the depth of the derivation of $\mathtt{D} \lhd \mathtt{C}$. Case analysis of the last rule in the derivation.

- Case S-Ref: Then D = C and the result follows.
- Case S-Trans: Then $\mathtt{D} \lhd \mathtt{T}$ and $\mathtt{T} \lhd \mathtt{C}$. By Lemma 7.2 we have that T is some class E. Then by induction we have *fields*(C) $\subseteq$ *fields*(E), and by induction again we have *fields*(E) $\subseteq$ *fields*(D). Then by transitivity of $\subseteq$ the result follows.
- Case S-Expand: Then D has the form $\mathtt{S}^{\mathtt{X}}$, which is a contradiction.
- Case S-Cls1: Then TT(D) = `class D extends C implements` $\overline{\mathtt{I}}$ $\{\overline{\mathtt{S}}\ \overline{\mathtt{g}}$; ...$\}$. By FieldsC we have *fields*(D) = $\overline{\mathtt{T}}\ \overline{\mathtt{f}}$, $\overline{\mathtt{S}}\ \overline{\mathtt{g}}$, so the result follows.
- Case S-Cls2: Then C is an interface, contradicting our initial assumption.
- Case S-Int: Then C is an interface, contradicting our initial assumption.
- Case S-Exp: Then C is an interface, contradicting our initial assumption.

Lemma 7.6 If $\mathtt{S} \lhd \mathtt{T}$ and *ftype*(f,T) = U, then *ftype*(f,S) = U.
Proof By induction on the depth of the derivation of $\mathtt{S} \lhd \mathtt{T}$. Case analysis of the last rule in the derivation.

- Case S-Ref: Then S = T and the result follows.
- Case S-Trans: Then $\mathtt{S} \lhd \mathtt{T}_0$ and $\mathtt{T}_0 \lhd \mathtt{T}$. By induction we have *ftype*(f,$\mathtt{T}_0$) = U, so by induction again also *ftype*(f,S) = U.

- Case S-EXPAND: Then $\mathtt{S}$ has the form $\mathtt{S}_0^{\mathtt{X}}$ and $\mathtt{T}$ has the form $\mathtt{T}_0^{\mathtt{X}}$ and $\mathtt{S}_0 \lhd \mathtt{T}_0$. Case analysis of the last rule in the derivation of *ftype*(f,T) = U:
  - Case FTYPE1: Then T has the form C, which contradicts our earlier assumption.
  - Case FTYPE2: Then *fields*(X) = $\overline{\mathtt{T}}\ \overline{\mathtt{f}} = \overline{\mathtt{v}}$ and $\mathtt{f} = \mathtt{f}_i$ and $\mathtt{U} = \mathtt{T}_i$. Then the result follows by FTYPE2.
  - Case FTYPE3: Then *fields*(X) = $\overline{\mathtt{T}}\ \overline{\mathtt{g}} = \overline{\mathtt{v}}$ and $\mathtt{f} \notin \overline{\mathtt{g}}$ and *ftype*(f,$\mathtt{T}_0$) = U. By induction we have *ftype*(f,$\mathtt{S}_0$) = U, and the result follows by FTYPE3.
- Case S-CLS1: Then S is a class D and T is a class C. Since *ftype*(f,T) = U, by FTYPE1 we have *fields*(C) = $\overline{\mathtt{T}}\ \overline{\mathtt{f}}$ and $\mathtt{f} = \mathtt{f}_i$ and $\mathtt{U} = \mathtt{T}_i$. By Lemma 7.5 we have *fields*(C) ⊆ *fields*(D), so the result follows by FTYPE1.
- Case S-CLS2: Then T is an interface, contradicting the fact that *ftype*(f,T) = U.
- Case S-INT: Then T is an interface, contradicting the fact that *ftype*(f,T) = U.
- Case S-EXP: Then T is an interface, contradicting the fact that *ftype*(f,T) = U.

Lemma 7.7 If *reallyImplements*(S, I) and *mtype*(m,I) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$, then *mtype*(m,S) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$.
Proof By induction on the depth of the derivation of *reallyImplements*(S, I). Since *reallyImplements*(S, I), by REALLYIMP we have TT(I) = `interface I extends` $\overline{\mathtt{J}}$ {$\overline{\mathtt{MH}}$}. We have two cases:

- Case $\mathtt{S}_0$ m($\overline{\mathtt{S}}\ \overline{\mathtt{x}}$); ∈ $\overline{\mathtt{MH}}$: Then by REALLYIMP we have *mtype*(m,S) = $\overline{\mathtt{U}} \rightarrow \mathtt{U}$ and *override*(m, I, $\overline{\mathtt{U}} \rightarrow \mathtt{U}$). Then by OVER1 we have that $\overline{\mathtt{U}} = \overline{\mathtt{T}}$ and U = T, so the result follows.
- Case m is not defined in $\overline{\mathtt{MH}}$: Since *mtype*(m,I) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$, by MTYPE-I2 we have *mtype*(m,$\mathtt{J}_i$) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$. Also, by REALLYIMP we have *reallyImplements*(S, $\mathtt{J}_i$). Therefore the result follows by induction.

Lemma 7.8 If $\mathtt{S}' \lhd \mathtt{S}$ and *mtype*(m,S) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$, then *mtype*(m,S′) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$.
Proof By induction on the depth of the derivation of $\mathtt{S}' \lhd \mathtt{S}$. Case analysis of the last rule in the derivation.

- Case S-REF: Then S′ = S and the result follows.
- Case S-TRANS: Then $\mathtt{S}' \lhd \mathtt{S}_0$ and $\mathtt{S}_0 \lhd \mathtt{S}$. By induction we have *mtype*(m,$\mathtt{S}_0$) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$, and by induction again we have *mtype*(m,S′) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$.
- Case S-EXPAND: Then S′ has the form $\mathtt{S}_0^{\mathtt{X}}$ and S has the form $\mathtt{S}_1^{\mathtt{X}}$ and $\mathtt{S}_0 \lhd \mathtt{S}_1$. Case analysis of the last rule in the derivation of *mtype*(m,S) = $\overline{\mathtt{T}} \rightarrow \mathtt{T}$:

- Case MTYPE-X1: Then $TT(\mathtt{X}) = \mathtt{expander\ X}\ \cdots\ \{\cdots\ \overline{\mathtt{M}}\}\ \overline{\mathtt{O}}$ and $\mathtt{T\ m(\overline{T}\ \overline{x})\ \{return\ t;\}} \in \overline{\mathtt{M}}$. Then the result follows by MTYPE-X1.
- Case MTYPE-X2: Then $TT(\mathtt{X}) = \mathtt{expander\ X}\ \cdots\ \{\cdots\ \overline{\mathtt{M}}\}\ \overline{\mathtt{O}}$ and m is not defined in $\overline{\mathtt{M}}$ and $mtype(\mathtt{m},\mathtt{S}_1) = \overline{\mathtt{T}}\rightarrow\mathtt{T}$. Since $\mathtt{S}_0 \lhd \mathtt{S}_1$, by induction also $mtype(\mathtt{m},\mathtt{S}_0) = \overline{\mathtt{T}}\rightarrow\mathtt{T}$. Then the result follows by MTYPE-X2.

- Case S-CLS1: Then $\mathtt{S}'$ is a class C and S is a class D and $TT(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements}\ \overline{\mathtt{I}}\ \{\cdots\ \overline{\mathtt{M}}\}$. We have two subcases:
  - Case $\mathtt{U\ m(\overline{U}\ \overline{x})\ \{return\ t;\}} \in \overline{\mathtt{M}}$: By COK we have $\overline{\mathtt{M}}$ OK in C, so by METHODOK we have $override(\mathtt{m}, \mathtt{D}, \overline{\mathtt{U}}\rightarrow\mathtt{U})$. Then by OVER1 we have that $\overline{\mathtt{U}} = \overline{\mathtt{T}}$ and $\mathtt{U} = \mathtt{T}$. Then the result follows by MTYPE-C1.
  - Case m is not defined in $\overline{\mathtt{M}}$: Then the result follows by MTYPE-C2.

- Case S-CLS2: Then $\mathtt{S}'$ is a class C and S is an interface $\mathtt{I}_i$ and $TT(\mathtt{C}) = \mathtt{class\ C\ extends\ D\ implements}\ \overline{\mathtt{I}}\ \{\cdots\ \overline{\mathtt{M}}\}$. By COK we have $reallyImplements(\mathtt{C},\mathtt{I}_i)$, so the result follows by Lemma 7.7.

- Case S-INT: Then $\mathtt{S}'$ is an interface I and S is an interface $\mathtt{J}_i$ and $TT(\mathtt{I}) = \mathtt{interface\ I\ extends}\ \overline{\mathtt{J}}\ \{\overline{\mathtt{MH}}\}$. By IOK we have $reallyImplements(\mathtt{I},\mathtt{J}_i)$, so the result follows by Lemma 7.7.

- Case S-EXP: Then $\mathtt{S}'$ has the form $\mathtt{U}^{\mathtt{X}}$ and S is an interface $\mathtt{I}_i$ and $TT(\mathtt{X}) = \mathtt{expander\ X\ of\ U\ implements}\ \overline{\mathtt{I}}\ \cdots$. By XOK we have $reallyImplements(\mathtt{U}^{\mathtt{X}},\mathtt{I}_i)$, so the result follows by Lemma 7.7.

Lemma 7.9 (Substitution) If $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t} : \mathtt{T}$ and $\Gamma \vdash \overline{\mathtt{s}} : \overline{\mathtt{S}}$ and $\overline{\mathtt{S}} \lhd \overline{\mathtt{T}}$, then $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t} : \mathtt{S}$ for some $\mathtt{S} \lhd \mathtt{T}$.
Proof By induction on the depth of the derivation of $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t} : \mathtt{T}$. Case analysis of the last rule in the derivation.

- Case T-VAR: Then t has the form x and $\mathtt{x}:\mathtt{T} \in \Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}}$. If $\mathtt{x} \notin \overline{\mathtt{x}}$ then we have $\mathtt{x}:\mathtt{T} \in \Gamma$, so by T-VAR we have $\Gamma \vdash \mathtt{x} : \mathtt{T}$. Since $\mathtt{x} \notin \overline{\mathtt{x}}$, we have $[\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{x} = \mathtt{x}$, and by S-REF we know $\mathtt{T} \lhd \mathtt{T}$, so the result follows. On the other hand, if $\mathtt{x} \in \overline{\mathtt{x}}$ then x has the form $\mathtt{x}_i$ and $\mathtt{T} = \mathtt{T}_i$ and $[\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{x} = \mathtt{s}_i$. We're given that $\Gamma \vdash \mathtt{s}_i : \mathtt{S}_i$ and $\mathtt{S}_i \lhd \mathtt{T}_i$, so the result follows.

- Case T-FIELD: Then t has the form s.f and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{s} : \mathtt{U}$ and $ftype(\mathtt{f},\mathtt{U}) = \mathtt{T}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{s} : \mathtt{U}_0$ and $\mathtt{U}_0 \lhd \mathtt{U}$. By Lemma 7.6 we have $ftype(\mathtt{f},\mathtt{U}_0) = \mathtt{T}$, so by T-FIELD also $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{s.f} : \mathtt{T}$, and by S-REF we have $\mathtt{T} \lhd \mathtt{T}$.

- Case T-INVK: Then t has the form $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}})$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t}_0 : \mathtt{T}_0$ and $mtype(\mathtt{m},\mathtt{T}_0) = \overline{\mathtt{U}}\rightarrow\mathtt{T}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \overline{\mathtt{t}} : \overline{\mathtt{U}_0}$ and $\overline{\mathtt{U}_0} \lhd \overline{\mathtt{U}}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t}_0 : \mathtt{T}_0'$ and $\mathtt{T}_0' \lhd \mathtt{T}_0$. By Lemma 7.8 we have $mtype(\mathtt{m},\mathtt{T}_0') = \overline{\mathtt{U}}\rightarrow\mathtt{T}$. Also by induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\overline{\mathtt{t}} : \overline{\mathtt{U}_0'}$ and $\overline{\mathtt{U}_0'} \lhd \overline{\mathtt{U}_0}$. Then by S-TRANS we have $\overline{\mathtt{U}_0'} \lhd \overline{\mathtt{U}}$. So by T-INVK we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) : \mathtt{T}$, and by S-REF we have $\mathtt{T} \lhd \mathtt{T}$.

- Case T-New: Then $\mathtt{t}$ has the form $\mathtt{new\ C(\overline{t})}$ and $\mathtt{T} = \mathtt{C}$ and *fields*$(\mathtt{C}) = \overline{\mathtt{U}}\ \overline{\mathtt{f}}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \overline{\mathtt{t}} : \overline{\mathtt{U_0}}$ and $\overline{\mathtt{U_0}} \lhd \overline{\mathtt{U}}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\overline{\mathtt{t}} : \overline{\mathtt{U}'_0}$ and $\overline{\mathtt{U}'_0} \lhd \overline{\mathtt{U_0}}$. Then by S-Trans we have $\overline{\mathtt{U}'_0} \lhd \overline{\mathtt{U}}$. So by T-New we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{new\ C(\overline{t})} : \mathtt{C}$, and by S-Ref we have $\mathtt{C} \lhd \mathtt{C}$.
- Case T-UCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T_0} \lhd \mathtt{T}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t_0} : \mathtt{T}'_0$ and $\mathtt{T}'_0 \lhd \mathtt{T_0}$. By S-Trans also $\mathtt{T}'_0 \lhd \mathtt{T}$, so by T-UCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$. Finally, by S-Ref we have $\mathtt{T} \lhd \mathtt{T}$.
- Case T-DCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T} \lhd \mathtt{T_0}$ and $\mathtt{T} \neq \mathtt{T_0}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t_0} : \mathtt{T}'_0$ and $\mathtt{T}'_0 \lhd \mathtt{T_0}$. If $\mathtt{T}'_0 \lhd \mathtt{T}$, then by T-UCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$. Otherwise if $\mathtt{T} \lhd \mathtt{T}'_0$, then by T-DCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$. Otherwise we have $\mathtt{T}'_0 \not\lhd \mathtt{T}$ and $\mathtt{T} \not\lhd \mathtt{T}'_0$, so by T-SCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$ and a *stupid warning* is generated. Finally, by S-Ref we have $\mathtt{T} \lhd \mathtt{T}$.
- Case T-SCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T_0} \not\lhd \mathtt{T}$ and $\mathtt{T} \not\lhd \mathtt{T_0}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{t_0} : \mathtt{T}'_0$ and $\mathtt{T}'_0 \lhd \mathtt{T_0}$. If $\mathtt{T}'_0 \lhd \mathtt{T}$, then by T-UCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$. Otherwise if $\mathtt{T} \lhd \mathtt{T}'_0$, then by S-Trans we have $\mathtt{T} \lhd \mathtt{T_0}$, which contradicts the fact that $\mathtt{T} \not\lhd \mathtt{T_0}$, so it is not possible that $\mathtt{T} \lhd \mathtt{T}'_0$. Otherwise we have $\mathtt{T}'_0 \not\lhd \mathtt{T}$ and $\mathtt{T} \not\lhd \mathtt{T}'_0$, so by T-SCast we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{(T)t_0} : \mathtt{T}$. Finally, by S-Ref we have $\mathtt{T} \lhd \mathtt{T}$.
- Case T-With: Then $\mathtt{t}$ has the form $\mathtt{s\ with\ X}$ and $\mathtt{T}$ has the form $\mathtt{U^X}$ and $TT(\mathtt{X}) = \mathtt{expander\ X\ of\ U_0} \cdots$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{s} : \mathtt{U}$ and $\mathtt{U} \lhd \mathtt{U_0}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{s} : \mathtt{U_1}$ and $\mathtt{U_1} \lhd \mathtt{U}$. Then by S-Trans we have $\mathtt{U_1} \lhd \mathtt{U_0}$, so by T-With we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{s\ with\ X} : \mathtt{U_1^X}$. Since $\mathtt{U_1} \lhd \mathtt{U}$, by S-Expand also $\mathtt{U_1^X} \lhd \mathtt{U^X}$.
- Case T-Peel: Then $\mathtt{t}$ has the form $\mathtt{peel\ s}$ and $\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{s} : \mathtt{T^X}$. By induction we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{s} : \mathtt{U}$ and $\mathtt{U} \lhd \mathtt{T^X}$. By Lemma 7.1, $\mathtt{U}$ has the form $\mathtt{U_0^X}$. Then by T-Peel we have $\Gamma \vdash [\overline{\mathtt{x}} \mapsto \overline{\mathtt{s}}]\mathtt{peel\ s} : \mathtt{U_0}$. Finally, by Lemma 7.4 we have $\mathtt{U_0} \lhd \mathtt{T}$.

Lemma 7.10 (Weakening) If $\Gamma \vdash \mathtt{t} : \mathtt{T}$ and $\mathtt{x} \notin \mathrm{dom}(\Gamma)$, then $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{t} : \mathtt{T}$.
Proof By induction on the depth of the derivation of $\Gamma \vdash \mathtt{t} : \mathtt{T}$. Case analysis of the last rule in the derivation.

- Case T-Var: Then $\mathtt{t}$ has the form $\mathtt{y}$ and $\mathtt{y}:\mathtt{T} \in \Gamma$. Since $\mathtt{x} \notin \mathrm{dom}(\Gamma)$, we have that $\mathtt{x} \neq \mathtt{y}$, so also $\mathtt{y}:\mathtt{T} \in \Gamma,\mathtt{x}:\mathtt{S}$. Therefore by T-Var we have $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{y} : \mathtt{T}$.
- Case T-Field: Then $\mathtt{t}$ has the form $\mathtt{s.f}$ and $\Gamma \vdash \mathtt{s} : \mathtt{U}$ and *ftype*$(\mathtt{f},\mathtt{U}) = \mathtt{T}$. By induction we have $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{s} : \mathtt{U}$, so by T-Field also $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{s.f} : \mathtt{T}$.
- Case T-Invk: Then $\mathtt{t}$ has the form $\mathtt{t_0.m(\overline{t})}$ and $\Gamma \vdash \mathtt{t_0} : \mathtt{T_0}$ and *mtype*$(\mathtt{m},\mathtt{T_0}) = \overline{\mathtt{T}} \to \mathtt{T}$ and $\Gamma \vdash \overline{\mathtt{t}} : \overline{\mathtt{S}}$ and $\overline{\mathtt{S}} \lhd \overline{\mathtt{T}}$. By induction we have $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\Gamma,\mathtt{x}:\mathtt{S} \vdash \overline{\mathtt{t}} : \overline{\mathtt{S}}$, so by T-Invk also $\Gamma,\mathtt{x}:\mathtt{S} \vdash \mathtt{t_0.m(\overline{t})} : \mathtt{T}$.

- Case T-New: Then $\mathtt{t}$ has the form $\mathtt{new\ C(\overline{t})}$ and $\mathtt{T} = \mathtt{C}$ and *fields*($\mathtt{C}$) = $\overline{\mathtt{T}}\ \overline{\mathtt{f}}$ and $\Gamma \vdash \overline{\mathtt{t}} : \overline{\mathtt{S}}$ and $\overline{\mathtt{S}} \lhd \overline{\mathtt{T}}$. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \overline{\mathtt{t}} : \overline{\mathtt{S}}$, so by T-New also $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{new\ C(\overline{t})} : \mathtt{C}$.

- Case T-UCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T_0} \lhd \mathtt{T}$. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{t_0} : \mathtt{T_0}$, so by T-UCast also $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{(T)t_0} : \mathtt{T}$.

- Case T-DCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T} \lhd \mathtt{T_0}$ and $\mathtt{T} \neq \mathtt{T_0}$. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{t_0} : \mathtt{T_0}$, so by T-DCast also $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{(T)t_0} : \mathtt{T}$.

- Case T-SCast: Then $\mathtt{t}$ has the form $\mathtt{(T)t_0}$ and $\Gamma \vdash \mathtt{t_0} : \mathtt{T_0}$ and $\mathtt{T_0} \not\lhd \mathtt{T}$ and $\mathtt{T} \not\lhd \mathtt{T_0}$ and a *stupid warning* is generated. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{t_0} : \mathtt{T_0}$, so by T-SCast also $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{(T)t_0} : \mathtt{T}$.

- Case T-With: Then $\mathtt{t}$ has the form $\mathtt{s\ with\ X}$ and $\mathtt{T}$ has the form $\mathtt{U^X}$ and $TT(\mathtt{X}) = \mathtt{expander\ X\ of\ U_0}\ \cdots$ and $\Gamma \vdash \mathtt{s} : \mathtt{U}$ and $\mathtt{U} \lhd \mathtt{U_0}$. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{s} : \mathtt{U}$, so the result follows by T-With.

- Case T-Peel: Then $\mathtt{t}$ has the form $\mathtt{peel\ s}$ and $\Gamma \vdash \mathtt{s} : \mathtt{T^X}$. By induction we have $\Gamma,\mathtt{x}{:}\mathtt{S} \vdash \mathtt{s} : \mathtt{T^X}$, so the result follows by T-Peel.

Lemma 7.11 If *mbody*($\mathtt{m}$,$\mathtt{C}$) = ($\overline{\mathtt{x}}$, $\mathtt{t}$) and *mtype*($\mathtt{m}$,$\mathtt{C}$) = $\overline{\mathtt{T}} \to \mathtt{T}$, then there exists a class $\mathtt{D}$ and a type $\mathtt{S}$ such that $\mathtt{C} \lhd \mathtt{D}$ and $\mathtt{S} \lhd \mathtt{T}$ and $\overline{\mathtt{x}}{:}\overline{\mathtt{T}},\mathtt{this}{:}\mathtt{D} \vdash \mathtt{t} : \mathtt{S}$.
Proof By induction on the depth of the derivation of *mbody*($\mathtt{m}$,$\mathtt{C}$) = ($\overline{\mathtt{x}}$, $\mathtt{t}$). Case analysis of the last rule in the derivation.

- Case MBody-C1: Then $TT(\mathtt{C}) = \mathtt{class\ C}\ \cdots\ \{\cdots\ \overline{\mathtt{M}}\}$ and $\mathtt{U\ m(\overline{U}\ \overline{x})\ \{return\ t;\}} \in \overline{\mathtt{M}}$. Since *mtype*($\mathtt{m}$,$\mathtt{C}$) = $\overline{\mathtt{T}} \to \mathtt{T}$, by MType-C1 we have that $\overline{\mathtt{U}} = \overline{\mathtt{T}}$ and $\mathtt{U} = \mathtt{T}$. By T-Class we have $\overline{\mathtt{M}}\ \mathtt{OK\ in\ C}$, so by MethodOK we have $\overline{\mathtt{x}}{:}\overline{\mathtt{T}},\mathtt{this}{:}\mathtt{C} \vdash \mathtt{t} : \mathtt{S}$ and $\mathtt{S} \lhd \mathtt{T}$. Finally, by S-Ref we have $\mathtt{C} \lhd \mathtt{C}$.

- Case MBody-C2: Then $TT(\mathtt{C}) = \mathtt{class\ C\ extends\ E\ implements}\ \overline{\mathtt{I}}\ \{\cdots\ \overline{\mathtt{M}}\}$ and $\mathtt{m}$ is not defined in $\overline{\mathtt{M}}$ and *mbody*($\mathtt{m}$,$\mathtt{C}$) = *mbody*($\mathtt{m}$,$\mathtt{E}$). Since *mtype*($\mathtt{m}$,$\mathtt{C}$) = $\overline{\mathtt{T}} \to \mathtt{T}$, by MType-C2 we have that *mtype*($\mathtt{m}$,$\mathtt{E}$) = $\overline{\mathtt{T}} \to \mathtt{T}$ as well. Therefore, by induction there exists a class $\mathtt{D}$ and a type $\mathtt{S}$ such that $\mathtt{E} \lhd \mathtt{D}$ and $\mathtt{S} \lhd \mathtt{T}$ and $\overline{\mathtt{x}}{:}\overline{\mathtt{T}},\mathtt{this}{:}\mathtt{D} \vdash \mathtt{t} : \mathtt{S}$. By S-Cls1 we have $\mathtt{C} \lhd \mathtt{E}$, so by S-Trans we have $\mathtt{C} \lhd \mathtt{D}$ and the result follows.

Lemma 7.12 If $TT(\mathtt{X}) = \mathtt{expander\ X}\ \cdots\ \{\cdots\ \overline{\mathtt{M'}}\}\ \overline{\mathtt{O}}$ and $\mathtt{of\ D}\ \{\overline{\mathtt{M}}\} \in \overline{\mathtt{O}}$ and $\mathtt{U\ m(\overline{U}\ \overline{x})\ \{return\ t;\}} \in \overline{\mathtt{M}}$, then $\mathtt{U\ m(\overline{U}\ \overline{y})\ \{return\ s;\}} \in \overline{\mathtt{M'}}$.
Proof By XOK, OOK, and OverrideOK we have *override*($\mathtt{m}$, $\mathtt{X}$, $\overline{\mathtt{U}} \to \mathtt{U}$). Then the result follows by Over2.

Lemma 7.13 If *mbody*($\mathtt{m}$,$\mathtt{X}$,$\mathtt{C}$,$\mathtt{D}$) = ($\overline{\mathtt{x}}$, $\mathtt{t}$) and *mtype*($\mathtt{m}$,$\mathtt{C^X}$) = $\overline{\mathtt{T}} \to \mathtt{T}$ and $\mathtt{C} \lhd \mathtt{D}$ and $TT(\mathtt{X}) = \mathtt{expander\ X\ of\ S_0}\ \cdots$ and $\mathtt{C} \lhd \mathtt{S_0}$, then there exists a type $\mathtt{T_0}$ and a type $\mathtt{S}$ such that $\mathtt{C^X} \lhd \mathtt{T_0^X}$ and $\mathtt{S} \lhd \mathtt{T}$ and $\overline{\mathtt{x}}{:}\overline{\mathtt{T}},\mathtt{this}{:}\mathtt{T_0^X} \vdash \mathtt{t} : \mathtt{S}$.

Proof By induction on the depth of the derivation of *mbody*(m,X,C,D) = ($\overline{\texttt{x}}$, t). Case analysis of the last rule in the derivation.

- Case MBody-X1: Then TT(X) = `expander X` ⋯ {⋯ $\overline{\texttt{M}'}$} $\overline{\texttt{O}}$ and `of D` {$\overline{\texttt{M}}$} ∈ $\overline{\texttt{O}}$ and `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{x}}$`) {return t;}` ∈ $\overline{\texttt{M}}$. Then by Lemma 7.12 we have `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{y}}$`) {return s;}` ∈ $\overline{\texttt{M}'}$. Then since *mtype*(m,$\texttt{C}^{\texttt{X}}$) = $\overline{\texttt{T}}$→T, by MType-X1 we have that $\overline{\texttt{U}}$ = $\overline{\texttt{T}}$ and U = T. By XOK, OOK, and OverrideOK we have `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{x}}$`) {return t;} OK in X,D`. Then by ExpMethodOK we have $\overline{\texttt{x}}$:$\overline{\texttt{T}}$,this:$\texttt{D}^{\texttt{X}}$ ⊢ t : S and S⊲T. Finally, since C⊲D, by S-Expand we have $\texttt{C}^{\texttt{X}}$⊲$\texttt{D}^{\texttt{X}}$.

- Case MBody-X2: Then TT(X) = `expander X` ⋯ {⋯ $\overline{\texttt{M}'}$} $\overline{\texttt{O}}$ and `of D` {$\overline{\texttt{M}}$} ∈ $\overline{\texttt{O}}$ and m is not defined in $\overline{\texttt{M}}$ and TT(D) = `class D extends E` ⋯ and *mbody*(m,X,C,E) = ($\overline{\texttt{x}}$, t). By S-Cls1 we have D⊲E, so by S-Trans we have C⊲E. Then the result follows by induction.

- Case MBody-X3: Then TT(X) = `expander X` ⋯ {⋯ $\overline{\texttt{M}'}$} $\overline{\texttt{O}}$ and C is not defined in $\overline{\texttt{O}}$ and TT(D) = `class D extends E` ⋯ and *mbody*(m,X,C,E) = ($\overline{\texttt{x}}$, t). By S-Cls1 we have D⊲E, so by S-Trans we have C⊲E. Then the result follows by induction.

- Case MBody-X4: Then TT(X) = `expander X of` $\texttt{S}_0$ {⋯ $\overline{\texttt{M}}$} $\overline{\texttt{O}}$ and `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{x}}$`) {return t;}` ∈ $\overline{\texttt{M}}$. Then since *mtype*(m,$\texttt{C}^{\texttt{X}}$) = $\overline{\texttt{T}}$→T, by MType-X1 we have that $\overline{\texttt{U}}$ = $\overline{\texttt{T}}$ and U = T. By XOK and ExpMethodOK we have $\overline{\texttt{x}}$:$\overline{\texttt{T}}$,this:$\texttt{S}_0^{\texttt{X}}$ ⊢ t : S and S⊲T. Finally, since C⊲$\texttt{S}_0$, by S-Expand we have $\texttt{C}^{\texttt{X}}$⊲$\texttt{S}_0^{\texttt{X}}$.

Theorem 7.1 (Type Preservation) If Γ ⊢ t : T and t ⟶ s, then there exists some type S such that Γ ⊢ s : S and S⊲T.

Proof By induction on the depth of the derivation of t ⟶ s. Case analysis of the last rule in the derivation.

- Case E-ProjNew: Then t has the form (`new C(`$\overline{\texttt{v}}$`)).`$\texttt{f}_i$ and s has the form $\texttt{v}_i$ and *fields*(C) = $\overline{\texttt{T}}$ $\overline{\texttt{f}}$. Since Γ ⊢ t : T, by T-Field and T-New we have that Γ ⊢ `new C(`$\overline{\texttt{v}}$`)` : C and Γ ⊢ $\texttt{v}_i$: $\texttt{S}_i$ and $\texttt{S}_i$⊲$\texttt{T}_i$ and *ftype*(f, C) = T. Then by FType1 we have T = $\texttt{T}_i$, so the result follows.

- Case E-ProjWith1: Then t has the form (`v with X).`$\texttt{f}_i$ and s has the form $\texttt{v}_i$ and *fields*(X) = $\overline{\texttt{T}}$ $\overline{\texttt{f}}$ = $\overline{\texttt{v}}$, so by FieldsX we have `expander X of S implements` $\overline{\texttt{I}}$ {$\overline{\texttt{T}}$ $\overline{\texttt{f}}$ = $\overline{\texttt{v}}$; $\overline{\texttt{M}}$} $\overline{\texttt{O}}$. Then by XOK we have • ⊢ $\texttt{v}_i$: $\texttt{S}_i$ and $\texttt{S}_i$⊲$\texttt{T}_i$, so by Lemma 7.10 also Γ ⊢ $\texttt{v}_i$: $\texttt{S}_i$. Since Γ ⊢ t : T, by T-Field and T-With we have Γ ⊢ `v with X` : $\texttt{U}^{\texttt{X}}$ and *ftype*(f, $\texttt{U}^{\texttt{X}}$) = T, so by FType2 we have T = $\texttt{T}_i$ and the result follows.

- Case E-ProjWith2: Then t has the form (`v with X).f` and s has the form `v.f` and *fields*(X) = $\overline{\texttt{T}}$ $\overline{\texttt{g}}$ = $\overline{\texttt{v}}$ and f ∉ $\overline{\texttt{g}}$. Since Γ ⊢ t : T, by T-Field and T-With we have Γ ⊢ `v with X` : $\texttt{U}^{\texttt{X}}$ and Γ ⊢ v : U and *ftype*(f, $\texttt{U}^{\texttt{X}}$) = T. Then by FType3 we have *ftype*(f, $\texttt{U}^{\texttt{X}}$) = *ftype*(f, U). Therefore by T-Field we have Γ ⊢ `v.f` : T, and by S-Ref we have T⊲T.

- Case E-InvkNew: Then $\texttt{t}$ has the form $\texttt{new C(}\overline{\texttt{v}}\texttt{).m(}\overline{\texttt{u}}\texttt{)}$ and $\texttt{s}$ has the form $[\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{new C(}\overline{\texttt{v}}\texttt{)}]\texttt{t}_0$ and $mbody(\texttt{m,C}) = (\overline{\texttt{x}}, \texttt{t}_0)$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{new C(}\overline{\texttt{v}}\texttt{)} : \texttt{S}'$ and $mtype(\texttt{m},\texttt{S}') = \overline{\texttt{T}}{\rightarrow}\texttt{T}$ and $\Gamma \vdash \overline{\texttt{u}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}{\lhd}\overline{\texttt{T}}$. By T-New we have that $\texttt{S}' = \texttt{C}$. Therefore by Lemma 7.11 there exists a class $\texttt{D}$ and a type $\texttt{U}$ such that $\texttt{C}{\lhd}\texttt{D}$ and $\texttt{U}{\lhd}\texttt{T}$ and $\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:D} \vdash \texttt{t}_0 : \texttt{U}$. Then by Lemma 7.10 also $\Gamma\texttt{,}\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:D} \vdash \texttt{t}_0 : \texttt{U}$, and by Lemma 7.9 we have $\Gamma \vdash [\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{new C(}\overline{\texttt{v}}\texttt{)}]\texttt{t}_0 : \texttt{S}$ and $\texttt{S}{\lhd}\texttt{U}$. Finally, by S-Trans we have $\texttt{S}{\lhd}\texttt{T}$.

- Case E-InvkWith1: Then $\texttt{t}$ has the form $\texttt{(v with X).m(}\overline{\texttt{u}}\texttt{)}$ and $\texttt{v}$ has the form new C($\overline{\texttt{v}}$) and $\texttt{s}$ has the form $[\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{(v with X)}]\texttt{t}_0$ and $mbody(\texttt{m,X,C,C}) = (\overline{\texttt{x}}, \texttt{t}_0)$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{v with X} : \texttt{S}'$ and $mtype(\texttt{m},\texttt{S}') = \overline{\texttt{T}}{\rightarrow}\texttt{T}$ and $\Gamma \vdash \overline{\texttt{u}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}{\lhd}\overline{\texttt{T}}$. By T-With and T-New we have that $\texttt{S}' = \texttt{C}^{\texttt{X}}$ and $\Gamma \vdash \texttt{v} : \texttt{C}$ and $TT(\texttt{X}) = \texttt{expander X of S}_0 \cdots$ and $\texttt{C}{\lhd}\texttt{S}_0$. Further, by S-Ref we have $\texttt{C}{\lhd}\texttt{C}$. Therefore by Lemma 7.13 there exists a type $\texttt{T}_0$ and a type $\texttt{U}$ such that $\texttt{C}^{\texttt{X}}{\lhd}\texttt{T}_0^{\texttt{X}}$ and $\texttt{U}{\lhd}\texttt{T}$ and $\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:T}_0^{\texttt{X}} \vdash \texttt{t}_0 : \texttt{U}$. Then by Lemma 7.10 also $\Gamma\texttt{,}\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:T}_0^{\texttt{X}} \vdash \texttt{t}_0 : \texttt{U}$, and by Lemma 7.9 we have that $\Gamma \vdash [\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{v with X}]\texttt{t}_0 : \texttt{S}$ and $\texttt{S}{\lhd}\texttt{U}$. Finally, by S-Trans we have $\texttt{S}{\lhd}\texttt{T}$.

- Case E-InvkWith2: Then $\texttt{t}$ has the form $\texttt{(v with X).m(}\overline{\texttt{u}}\texttt{)}$ and $\texttt{v}$ has the form $\texttt{v' with X'}$ and $\texttt{s}$ has the form $[\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{(v with X)}]\texttt{t}_0$ and $mbody(\texttt{m,X,Object,Object}) = (\overline{\texttt{x}}, \texttt{t}_0)$. Case analysis of the last rule in the derivation of $mbody(\texttt{m,X,Object,Object}) = (\overline{\texttt{x}}, \texttt{t}_0)$:

  - Case MBody-X1: Then $TT(\texttt{X}) = \texttt{expander X of S}_0 \cdots \overline{\texttt{O}}$ and $\texttt{of Object} \cdots \in \overline{\texttt{O}}$. By a sanity condition on FeJ programs we have that $\texttt{S}_0 \neq \texttt{Object}$, and by OOK we have $\texttt{Object}{\lhd}\texttt{S}_0$. Then we have a contradiction by Lemma 7.3.
  - Case MBody-X2: Then $\texttt{Object} \in dom(TT)$, which contradicts an assumption about FeJ programs.
  - Case MBody-X3: Then $\texttt{Object} \in dom(TT)$, which contradicts an assumption about FeJ programs.
  - Case MBody-X4: Then $TT(\texttt{X}) = \texttt{expander X of S}_0\ \{\cdots\ \overline{\texttt{M}}\}\ \overline{\texttt{O}}$ and $\texttt{U m(}\overline{\texttt{U}}\ \overline{\texttt{x}}\texttt{) \{return t}_0\texttt{;\}} \in \overline{\texttt{M}}$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{v with X} : \texttt{S}'$ and $mtype(\texttt{m},\texttt{S}') = \overline{\texttt{T}}{\rightarrow}\texttt{T}$ and $\Gamma \vdash \overline{\texttt{u}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}{\lhd}\overline{\texttt{T}}$. By T-With we have $\texttt{S}' = \texttt{S}_1^{\texttt{X}}$ and $\Gamma \vdash \texttt{v} : \texttt{S}_1$ and $\texttt{S}_1{\lhd}\texttt{S}_0$. Then since $mtype(\texttt{m},\texttt{S}_1^{\texttt{X}}) = \overline{\texttt{T}}{\rightarrow}\texttt{T}$, by MType-X1 we have that $\overline{\texttt{U}} = \overline{\texttt{T}}$ and $\texttt{U} = \texttt{T}$. By XOK and ExpMethodOK we have $\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:S}_0^{\texttt{X}} \vdash \texttt{t}_0 : \texttt{U}_0$ and $\texttt{U}_0{\lhd}\texttt{T}$. Also, since $\texttt{S}_1{\lhd}\texttt{S}_0$, by S-Expand we have $\texttt{S}_1^{\texttt{X}}{\lhd}\texttt{S}_0^{\texttt{X}}$.

    Therefore, by Lemma 7.10 we have $\Gamma\texttt{,}\overline{\texttt{x}}\texttt{:}\overline{\texttt{T}}\texttt{,this:S}_0^{\texttt{X}} \vdash \texttt{t}_0 : \texttt{U}_0$ and by Lemma 7.9 we have that $\Gamma \vdash [\overline{\texttt{x}} \mapsto \overline{\texttt{u}}\texttt{,this} \mapsto \texttt{v with X}]\texttt{t}_0 : \texttt{S}$ and $\texttt{S}{\lhd}\texttt{U}_0$. Finally, by S-Trans we have $\texttt{S}{\lhd}\texttt{T}$.

- Case E-InvkWith3: Then $\texttt{t}$ has the form $\texttt{(v with X).m}(\overline{\texttt{u}})$ and $\texttt{s} = \texttt{v.m}(\overline{\texttt{u}})$ and $TT(\texttt{X}) = \texttt{expander X of } \texttt{S}_0\ \{\cdots\ \overline{\texttt{M}}\}\ \overline{\texttt{O}}$ and $\texttt{m}$ is not defined in $\overline{\texttt{M}}$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{v with X} : \texttt{S}'$ and $\mathit{mtype}(\texttt{m},\texttt{S}') = \overline{\texttt{T}}\rightarrow\texttt{T}$ and $\Gamma \vdash \overline{\texttt{u}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}\lhd\overline{\texttt{T}}$. By T-With, $\texttt{S}'$ has the form $\texttt{S}_1^{\texttt{X}}$ and $\Gamma \vdash \texttt{v} : \texttt{S}_1$. Since $\mathit{mtype}(\texttt{m},\texttt{S}') = \overline{\texttt{T}}\rightarrow\texttt{T}$, by MType-X2 we have $\mathit{mtype}(\texttt{m},\texttt{S}_1) = \overline{\texttt{T}}\rightarrow\texttt{T}$. Therefore, by T-Invk we have $\Gamma \vdash \texttt{v.m}(\overline{\texttt{u}}) : T$. Finally, by S-Ref we have $\texttt{T}\lhd\texttt{T}$.

- Case E-CastVal: Then $\texttt{t}$ has the form $\texttt{(T}_0\texttt{)(v)}$ and $\texttt{s}$ has the form $\texttt{v}$ and $\bullet \vdash \texttt{v} : \texttt{S}_0$ and $\texttt{S}_0\lhd\texttt{T}_0$. Then by Lemma 7.10 also $\Gamma \vdash \texttt{v} : \texttt{S}_0$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-UCast, T-DCast, and T-SCast we have that $\texttt{T} = \texttt{T}_0$, so the result follows.

- Case E-Field: Then $\texttt{t}$ has the form $\texttt{t}_1\texttt{.f}$ and $\texttt{s}$ has the form $\texttt{t}_2\texttt{.f}$ and $\texttt{t}_1 \longrightarrow \texttt{t}_2$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Field we have $\Gamma \vdash \texttt{t}_1 : \texttt{T}_1$ and $\mathit{ftype}(\texttt{f},\texttt{T}_1) = \texttt{T}$. By induction, there exists some type $\texttt{T}_2$ such that $\Gamma \vdash \texttt{t}_2 : \texttt{T}_2$ and $\texttt{T}_2\lhd\texttt{T}_1$. Then by Lemma 7.6 also $\mathit{ftype}(\texttt{f},\texttt{T}_2) = \texttt{T}$. Therefore, by T-Field we have $\Gamma \vdash \texttt{t}_2\texttt{.f} : \texttt{T}$, and by S-Ref we have $\texttt{T}\lhd\texttt{T}$.

- Case E-Invk-Recv: Then $\texttt{t}$ has the form $\texttt{s}_1\texttt{.m}(\overline{\texttt{t}})$ and $\texttt{s}$ has the form $\texttt{s}_2\texttt{.m}(\overline{\texttt{t}})$ and $\texttt{s}_1 \longrightarrow \texttt{s}_2$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{s}_1 : \texttt{S}'$ and $\mathit{mtype}(\texttt{m},\texttt{S}') = \overline{\texttt{T}}\rightarrow\texttt{T}$ and $\Gamma \vdash \overline{\texttt{t}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}\lhd\overline{\texttt{T}}$. By induction we have $\Gamma \vdash \texttt{s}_2 : \texttt{S}''$ and $\texttt{S}''\lhd\texttt{S}'$. Then by Lemma 7.8 we have $\mathit{mtype}(\texttt{m},\texttt{S}'') = \overline{\texttt{T}}\rightarrow\texttt{T}$. Then by T-Invk we have $\Gamma \vdash \texttt{s}_2\texttt{.m}(\overline{\texttt{t}}) : \texttt{T}$ and by S-Ref we have $\texttt{T}\lhd\texttt{T}$.

- Case E-Invk-Arg: Then $\texttt{t}$ has the form $\texttt{v.m}(\overline{\texttt{v}},\texttt{s}_1,\overline{\texttt{s}_0})$ and $\texttt{s}$ has the form $\texttt{v.m}(\overline{\texttt{v}},\texttt{s}_2,\overline{\texttt{s}_0})$ and $\texttt{s}_1 \longrightarrow \texttt{s}_2$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-Invk we have $\Gamma \vdash \texttt{v} : \texttt{S}'$ and $\mathit{mtype}(\texttt{m},\texttt{S}') = \overline{\texttt{T}}\rightarrow\texttt{T}$ and $\overline{\texttt{v}},\texttt{s}_1,\overline{\texttt{s}_0} = \overline{\texttt{t}}$ and $\Gamma \vdash \overline{\texttt{t}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}\lhd\overline{\texttt{T}}$. Assume that $\texttt{s}_1$ is the ith element of $\overline{\texttt{t}}$. By induction we have that $\Gamma \vdash \texttt{s}_2 : \texttt{S}_i'$ and $\texttt{S}_i'\lhd\texttt{S}_i$. Then by S-Trans also $\texttt{S}_i'\lhd\texttt{T}_i$, so by T-Invk we have $\Gamma \vdash \texttt{v.m}(\overline{\texttt{v}},\texttt{s}_2,\overline{\texttt{s}_0}) : \texttt{T}$ and by S-Ref we have $\texttt{T}\lhd\texttt{T}$.

- Case E-New-Arg: Then $\texttt{t}$ has the form $\texttt{new C}(\overline{\texttt{v}},\texttt{s}_1,\overline{\texttt{s}_0})$ and $\texttt{s}$ has the form $\texttt{new C}(\overline{\texttt{v}},\texttt{s}_2,\overline{\texttt{s}_0})$ and $\texttt{s}_1 \longrightarrow \texttt{s}_2$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-New we have $\mathit{fields}(\texttt{C}) = \overline{\texttt{T}}\ \overline{\texttt{f}}$ and $\overline{\texttt{v}},\texttt{s}_1,\overline{\texttt{s}_0} = \overline{\texttt{t}}$ and $\Gamma \vdash \overline{\texttt{t}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}}\lhd\overline{\texttt{T}}$ and $\texttt{T} = \texttt{C}$. Assume that $\texttt{s}_1$ is the ith element of $\overline{\texttt{t}}$. By induction we have that $\Gamma \vdash \texttt{s}_2 : \texttt{S}_i'$ and $\texttt{S}_i'\lhd\texttt{S}_i$. Then by S-Trans also $\texttt{S}_i'\lhd\texttt{T}_i$, so by T-New we have $\Gamma \vdash \texttt{new C}(\overline{\texttt{v}},\texttt{s}_2,\overline{\texttt{s}_0}) : \texttt{C}$ and by S-Ref we have $\texttt{C}\lhd\texttt{C}$.

- Case E-Cast: Then $\texttt{t}$ has the form $\texttt{(T}_0\texttt{)s}_1$ and $\texttt{s}$ has the form $\texttt{(T}_0\texttt{)s}_2$ and $\texttt{s}_1 \longrightarrow \texttt{s}_2$. There are three subcases, depending on the last rule in the derivation of $\Gamma \vdash \texttt{t} : \texttt{T}$.

  - Case T-UCast: Then $\Gamma \vdash \texttt{s}_1 : \texttt{S}_0$ and $\texttt{S}_0\lhd\texttt{T}_0$ and $\texttt{T} = \texttt{T}_0$. By induction we have $\Gamma \vdash \texttt{s}_2 : \texttt{S}_0'$ and $\texttt{S}_0'\lhd\texttt{S}_0$. Then by S-Trans also $\texttt{S}_0'\lhd\texttt{T}_0$, so by T-UCast we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$ and by S-Ref we have $\texttt{T}_0\lhd\texttt{T}_0$.

– Case T-DCAST: Then $\Gamma \vdash \texttt{s}_1 : \texttt{S}_0$ and $\texttt{T}_0 \lhd \texttt{S}_0$ and $\texttt{T}_0 \neq \texttt{S}_0$ and and $\texttt{T} = \texttt{T}_0$. By induction we have $\Gamma \vdash \texttt{s}_2 : \texttt{S}'_0$ and $\texttt{S}'_0 \lhd \texttt{S}_0$. If $\texttt{S}'_0 \lhd \texttt{T}_0$ then by T-UCAST we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$. Otherwise, if $\texttt{T}_0 \lhd \texttt{S}'_0$ then by T-DCAST we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$. Otherwise we have $\texttt{S}'_0 \not\lhd \texttt{T}_0$ and $\texttt{T}_0 \not\lhd \texttt{S}'_0$, so by T-SCAST we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$ along with the generation of a *stupid warning*. Finally, by S-REF we have $\texttt{T}_0 \lhd \texttt{T}_0$.

– Case T-SCAST: Then $\Gamma \vdash \texttt{s}_1 : \texttt{S}_0$ and $\texttt{S}_0 \not\lhd \texttt{T}_0$ and $\texttt{T}_0 \not\lhd \texttt{S}_0$ and a *stupid warning* is generated and $\texttt{T} = \texttt{T}_0$. By induction we have $\Gamma \vdash \texttt{s}_2 : \texttt{S}'_0$ and $\texttt{S}'_0 \lhd \texttt{S}_0$. If $\texttt{S}'_0 \lhd \texttt{T}_0$ then by T-UCAST we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$. Otherwise, if $\texttt{T}_0 \lhd \texttt{S}'_0$ then by S-TRANS also $\texttt{T}_0 \lhd \texttt{S}_0$, contradicting the fact that $\texttt{T}_0 \not\lhd \texttt{S}_0$, so it is not possible that $\texttt{T}_0 \lhd \texttt{S}'_0$. Otherwise we have $\texttt{S}'_0 \not\lhd \texttt{T}_0$ and $\texttt{T}_0 \not\lhd \texttt{S}'_0$, so by T-SCAST we have $\Gamma \vdash \texttt{(T}_0\texttt{)s}_2 : \texttt{T}_0$. Finally, by S-REF we have $\texttt{T}_0 \lhd \texttt{T}_0$.

- Case E-WITH: Then `t` has the form $\texttt{t}_0$ `with X` and `s` has the form $\texttt{s}_0$ `with X` and $\texttt{t}_0 \longrightarrow \texttt{s}_0$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-WITH `T` has the form $\texttt{U}^\texttt{X}$ and $TT(\texttt{X}) = \texttt{expander X of U}_0 \cdots$ and $\Gamma \vdash \texttt{t}_0 : \texttt{U}$ and $\texttt{U} \lhd \texttt{U}_0$. By induction we have $\Gamma \vdash \texttt{s}_0 : \texttt{U}_1$ and $\texttt{U}_1 \lhd \texttt{U}$, so by S-TRANS also $\texttt{U}_1 \lhd \texttt{U}_0$. Therefore by T-WITH we have $\Gamma \vdash \texttt{s} : \texttt{U}_1^\texttt{X}$. Finally, since $\texttt{U}_1 \lhd \texttt{U}$, by S-EXPAND also $\texttt{U}_1^\texttt{X} \lhd \texttt{U}^\texttt{X}$.

- Case E-PEEL: Then `t` has the form `peel` $\texttt{t}_0$ and `s` has the form `peel` $\texttt{s}_0$ and $\texttt{t}_0 \longrightarrow \texttt{s}_0$. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-PEEL $\Gamma \vdash \texttt{t}_0 : \texttt{T}^\texttt{X}$. By induction we have $\Gamma \vdash \texttt{s}_0 : \texttt{U}$ and $\texttt{U} \lhd \texttt{T}^\texttt{X}$, so by Lemma 7.1, `U` has the form $\texttt{S}^\texttt{X}$. Therefore by T-PEEL we have $\Gamma \vdash \texttt{s} : \texttt{S}$. Finally, by Lemma 7.4 we have $\texttt{S} \lhd \texttt{T}$.

- Case E-PEELWITH: Then `t` has the form `peel (v with X)` and `s` = `v`. Since $\Gamma \vdash \texttt{t} : \texttt{T}$, by T-PEEL we have $\Gamma \vdash \texttt{v with X} : \texttt{T}^\texttt{Y}$. Then by T-WITH we have $\texttt{X} = \texttt{Y}$ and $\Gamma \vdash \texttt{v} : \texttt{T}$. Finally, by S-REF we have $\texttt{T} \lhd \texttt{T}$.

7.2 Progress

Lemma 7.14 (Canonical Forms) If $\Gamma \vdash \texttt{v} : \texttt{T}^\texttt{X}$ then `v` has the form `v' with X`.
Proof Case analysis of the last rule in the derivation of $\Gamma \vdash \texttt{v} : \texttt{T}^\texttt{X}$. By the syntax of values, there are only two cases:

- Case T-NEW: Then $\texttt{T}^\texttt{X}$ is a class `C`, which is a contradiction.
- Case T-WITH: Then the result follows.

Lemma 7.15 If $mtype(\texttt{m},\texttt{C}) = \overline{\texttt{T}} \rightarrow \texttt{T}$, then there exist $\overline{\texttt{x}}$ and `t` such that $mbody(\texttt{m},\texttt{C}) = (\overline{\texttt{x}}, \texttt{t})$.
Proof By induction on the depth of the derivation of $mtype(\texttt{m},\texttt{C}) = \overline{\texttt{T}} \rightarrow \texttt{T}$. Case analysis of the last rule in the derivation:

- Case MTYPE-C1: Then $TT(\texttt{C}) = \texttt{class C} \cdots \{\cdots \overline{\texttt{M}}\}$ and $\texttt{T m}(\overline{\texttt{T}}\ \overline{\texttt{x}})\ \{\texttt{return t;}\} \in \overline{\texttt{M}}$, and the result follows by MBODY-C1.

- Case MType-C2: Then TT(C) = `class C extends D implements` $\overline{\texttt{I}}$ {$\cdots$ $\overline{\texttt{M}}$} and `m` is not defined in $\overline{\texttt{M}}$ and *mtype*(m,D) = ($\overline{\texttt{x}}$, t). By induction there exist $\overline{\texttt{x}}$ and `t` such that *mbody*(m,D) = ($\overline{\texttt{x}}$, t), and the result follows by MBody-C2.

Lemma 7.16 If TT(X) = `expander X of` $\texttt{S}_0$ $\cdots$ {$\cdots$ $\overline{\texttt{M}}$} $\overline{\texttt{O}}$ and `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{y}}$`) {return s;}` $\in \overline{\texttt{M}}$, then there exist $\overline{\texttt{x}}$ and `t` such that *mbody*(m,X,C,D) = ($\overline{\texttt{x}}$, t).
Proof By strong induction on the number classes `E` such that D$\triangleleft$E. There are a number of cases:

- Case `of D` {$\overline{\texttt{M'}}$} $\in \overline{\texttt{O}}$ and `U' m(`$\overline{\texttt{U'}}$ $\overline{\texttt{x}}$`) {return t;}` $\in \overline{\texttt{M'}}$: Then the result follows by MBody-X1.

- Case `of D` {$\overline{\texttt{M'}}$} $\in \overline{\texttt{O}}$ and `m` is not defined in $\overline{\texttt{M'}}$: We have two subcases. First suppose that TT(D) = `class D extends E` $\cdots$. By induction there exist $\overline{\texttt{x}}$ and `t` such that *mbody*(m,X,C,E) = ($\overline{\texttt{x}}$, t), and the result follows by MBody-X2. Second, suppose D $\notin$ *dom*(TT). Then D = `Object`. Since we're given that TT(X) = `expander X of` $\texttt{S}_0$ $\cdots$ {$\cdots$ $\overline{\texttt{M}}$} $\overline{\texttt{O}}$ and `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{y}}$`) {return s;}` $\in \overline{\texttt{M}}$, the result follows by MBody-X4.

- Case `D` is not defined in $\overline{\texttt{O}}$: We have two subcases. First suppose that TT(D) = `class D extends E` $\cdots$. By induction there exist $\overline{\texttt{x}}$ and `t` such that *mbody*(m,X,C,E) = ($\overline{\texttt{x}}$, t), and the result follows by MBody-X3. Second, suppose D $\notin$ *dom*(TT). Then D = `Object`. Since we're given that TT(X) = `expander X of` $\texttt{S}_0$ $\cdots$ {$\cdots$ $\overline{\texttt{M}}$} $\overline{\texttt{O}}$ and `U m(`$\overline{\texttt{U}}$ $\overline{\texttt{y}}$`) {return s;}` $\in \overline{\texttt{M}}$, the result follows by MBody-X4.

Theorem 7.2 (Progress) If $\bullet \vdash$ t : T, then either `t` is a value, `t` contains a subexpression of the form `(U)(v)` where $\bullet \vdash$ v : S and S $\not\triangleleft$ U, or there exists some term `s` such that t $\longrightarrow$ s.
Proof By induction on the depth of the derivation of $\bullet \vdash$ t : T. Case analysis of the last rule in the derivation.

- Case T-Var: Then `t` has the form `x` and x:T $\in \bullet$, which is a contradiction. Therefore, T-Var cannot be the last rule in the derivation.

- Case T-Field: Then `t` has the form $\texttt{t}_0$.f and $\bullet \vdash \texttt{t}_0 : \texttt{T}_0$ and *ftype*(f,$\texttt{T}_0$) = T. By induction, there are three subcases.
  - Case $\texttt{t}_0$ is a value. Case analysis on the form of $\texttt{t}_0$.
    * Case $\texttt{t}_0$ has the form `new` $\texttt{C}_0$($\overline{\texttt{v}}$): Since $\bullet \vdash \texttt{t}_0 : \texttt{T}_0$, by T-New $\texttt{T}_0$ is $\texttt{C}_0$ and *fields*($\texttt{C}_0$) = $\overline{\texttt{S}}$ $\overline{\texttt{f}}$ and $\overline{\texttt{v}}$ has the same length as $\overline{\texttt{f}}$. Then since *ftype*(f,$\texttt{T}_0$) = T, by FType1 we have that f = $\texttt{f}_i$ and T = $\texttt{S}_i$. Then by E-ProjNew we have $\texttt{t}_0.\texttt{f}_i \longrightarrow \texttt{v}_i$.
    * Case $\texttt{t}_0$ has the form `v with X`: Then TT(X) = `expander X` $\cdots$ {$\overline{\texttt{S}}$ $\overline{\texttt{f}}$ = $\overline{\texttt{v}}$; $\overline{\texttt{M}}$} $\overline{\texttt{O}}$, and by FieldsX we have *fields*(X) = $\overline{\texttt{S}}$ $\overline{\texttt{f}}$ = $\overline{\texttt{v}}$. There are two subcases. First suppose that

$\mathtt{f} \in \overline{\mathtt{f}}$, so $\mathtt{f}$ has the form $\mathtt{f}_i$. Then by E-ProjWith1 we have $\mathtt{t}_0\mathtt{.f}_i \longrightarrow \mathtt{v}_i$. Now suppose that $\mathtt{f} \notin \overline{\mathtt{f}}$. Then by E-ProjWith2 we have $\mathtt{t}_0\mathtt{.f}_i \longrightarrow \mathtt{v.f}$.

- Case $\mathtt{t}_0$ contains a subexpression of the form $\mathtt{(U)(v)}$ where $\bullet \vdash \mathtt{v} : \mathtt{S}$ and $\mathtt{S} \not\triangleleft \mathtt{U}$. Then so does $\mathtt{t}$.
- Case there exists some term $\mathtt{s}_0$ such that $\mathtt{t}_0 \longrightarrow \mathtt{s}_0$. Then by E-Field we have $\mathtt{t}_0\mathtt{.f} \longrightarrow \mathtt{s}_0\mathtt{.f}$.

- Case T-Invk: Then $\mathtt{t}$ has the form $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}})$ and $\bullet \vdash \mathtt{t}_0 : \mathtt{T}_0$ and *mtype*($\mathtt{m}$,$\mathtt{T}_0$) $= \overline{\mathtt{T}}\rightarrow\mathtt{T}$ and $\bullet \vdash \overline{\mathtt{t}} : \overline{\mathtt{S}}$ and $\overline{\mathtt{S}}\triangleleft\overline{\mathtt{T}}$. By induction, there are three subcases.
  - Case $\mathtt{t}_0$ is a value. By induction, there are three subcases.
    - Case all terms in $\overline{\mathtt{t}}$ are values. We do a case analysis on the form of $\mathtt{t}_0$:
      - Case $\mathtt{t}_0$ has the form $\mathtt{new\ C}(\overline{\mathtt{v}})$: Then by T-New, $\mathtt{T}_0 = \mathtt{C}$. Then by Lemma 7.15 there exist $\overline{\mathtt{x}}$ and $\mathtt{s}_0$ such that *mbody*($\mathtt{m}$,$\mathtt{C}$) $= (\overline{\mathtt{x}}, \mathtt{s}_0)$. Then by E-InvkNew we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow [\overline{\mathtt{x}} \mapsto \overline{\mathtt{t}},\ \mathtt{this} \mapsto \mathtt{new\ C}_0(\overline{\mathtt{v}})]\mathtt{s}_0$.
      - Case $\mathtt{t}_0$ has the form $\mathtt{v\ with\ X}$: Then $TT(\mathtt{X})$ has the form $\mathtt{expander\ X\ of\ S}_0 \cdots \{\cdots \overline{\mathtt{M}}\}\ \overline{\mathtt{O}}$. First suppose that $\mathtt{m}$ is not defined in $\overline{\mathtt{M}}$. Then by E-InvkWith3 we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow \mathtt{v.m}(\overline{\mathtt{t}})$. Otherwise, we have that $\mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{y}})\ \{\mathtt{return\ s;}\} \in \overline{\mathtt{M}}$, and we do a case analysis on the form of $\mathtt{v}$.
        Suppose $\mathtt{v}$ has the form $\mathtt{(new\ C}(\overline{\mathtt{v}})\mathtt{)}$. By Lemma 7.16 there exist $\overline{\mathtt{x}}$ and $\mathtt{s}_0$ such that *mbody*($\mathtt{m}$,$\mathtt{X}$,$\mathtt{C}$,$\mathtt{C}$) $= (\overline{\mathtt{x}}, \mathtt{s}_0)$. Then by E-InvkWith1 we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow [\overline{\mathtt{x}} \mapsto \overline{\mathtt{t}},\ \mathtt{this} \mapsto \mathtt{t}_0]\mathtt{s}_0$.
        Finally, suppose $\mathtt{v}$ has the form $\mathtt{v'\ with\ X}$. Since $\mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{y}})\ \{\mathtt{return\ s;}\} \in \overline{\mathtt{M}}$, by MBody-X4 we have *mbody*($\mathtt{m}$,$\mathtt{X}$,$\mathtt{Object}$,$\mathtt{Object}$) $= (\overline{\mathtt{y}}, \mathtt{s})$. Then by E-InvkWith2 we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow [\overline{\mathtt{y}} \mapsto \overline{\mathtt{t}},\ \mathtt{this} \mapsto \mathtt{t}_0]\mathtt{s}$.
    - Case some term in $\overline{\mathtt{t}}$ contains a subexpression of the form $\mathtt{(U)(v)}$ where $\bullet \vdash \mathtt{v} : \mathtt{S}$ and $\mathtt{S} \not\triangleleft \mathtt{U}$. Then so does $\mathtt{t}$.
    - Case no term in $\overline{\mathtt{t}}$ contains a subexpression of the form $\mathtt{(U)(new\ C}(\overline{\mathtt{u}})\mathtt{)}$ where $\mathtt{C} \not\triangleleft \mathtt{U}$. Further, there is some $\mathtt{t}_i \in \overline{\mathtt{t}}$ for which there exists a term $\mathtt{s}_i$ such that $\mathtt{t}_i \longrightarrow \mathtt{s}_i$. Further, all $\mathtt{t}_j$ such that $1 \leq j < i$ are values. Then by E-Invk-Arg we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow \mathtt{t}_0\mathtt{.m}(\mathtt{t}_1,\ldots,\mathtt{t}_{i-1},\mathtt{s}_i,\mathtt{t}_{i+1},\ldots,\mathtt{t}_n)$.
  - Case $\mathtt{t}_0$ contains a subexpression of the form $\mathtt{(U)(v)}$ where $\bullet \vdash \mathtt{v} : \mathtt{S}$ and $\mathtt{S} \not\triangleleft \mathtt{U}$. Then so does $\mathtt{t}$.
  - Case there exists some term $\mathtt{s}_0$ such that $\mathtt{t}_0 \longrightarrow \mathtt{s}_0$. Then by E-Invk-Recv we have $\mathtt{t}_0\mathtt{.m}(\overline{\mathtt{t}}) \longrightarrow \mathtt{s}_0\mathtt{.m}(\overline{\mathtt{t}})$.

- Case T-New: Then $\texttt{t}$ has the form $\texttt{new C}_0(\overline{\texttt{t}})$ and $\texttt{T}$ is $\texttt{C}_0$ and *fields*$(\texttt{C}_0)$ $= \overline{\texttt{T}}\ \overline{\texttt{f}}$ and $\bullet \vdash \overline{\texttt{t}} : \overline{\texttt{S}}$ and $\overline{\texttt{S}} \lhd \overline{\texttt{T}}$. By induction, there are three subcases.
  - Case all terms in $\overline{\texttt{t}}$ are values. Then also $\texttt{t}$ is a value.
  - Case some term in $\overline{\texttt{t}}$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$. Then so does $\texttt{t}$.
  - Case no term in $\overline{\texttt{t}}$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$. Further, there is some $\texttt{t}_i \in \overline{\texttt{t}}$ for which there exists a term $\texttt{s}_i$ such that $\texttt{t}_i \longrightarrow \texttt{s}_i$. Further, all $\texttt{t}_j$ such that $1 \leq j < i$ are values. Then by E-New-Arg we have $\texttt{new C}_0(\overline{\texttt{t}}) \longrightarrow \texttt{new C}_0(\texttt{t}_1,\ldots,\texttt{t}_{i-1},\texttt{s}_i,\texttt{t}_{i+1},\ldots,\texttt{t}_n)$.
- Case T-UCast: Then $\texttt{t}$ has the form $\texttt{(T)t}_0$ and $\bullet \vdash \texttt{t}_0 : \texttt{S}_0$ and $\texttt{S}_0 \lhd \texttt{T}$. By induction, there are three subcases.
  - Case $\texttt{t}_0$ is a value. Then by E-CastNew we have $\texttt{(T)t}_0 \longrightarrow \texttt{t}_0$.
  - Case $\texttt{t}_0$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$. Then so does $\texttt{t}$.
  - Case there exists some term $\texttt{s}_0$ such that $\texttt{t}_0 \longrightarrow \texttt{s}_0$. Then by E-Cast we have $\texttt{(T)t}_0 \longrightarrow \texttt{(T)s}_0$.
- Case T-DCast: Then $\texttt{t}$ has the form $\texttt{(T)t}_0$ and $\bullet \vdash \texttt{t}_0 : \texttt{S}_0$ and $\texttt{T} \lhd \texttt{S}_0$ and $\texttt{T} \neq \texttt{S}_0$. By induction, there are three subcases.
  - Case $\texttt{t}_0$ is a value. If $\texttt{S}_0 \lhd \texttt{T}$ then by E-CastNew we have $\texttt{(T)t}_0 \longrightarrow \texttt{t}_0$. Otherwise $\texttt{S}_0 \not\lhd \texttt{T}$, so $\texttt{t}$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$.
  - Case $\texttt{t}_0$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$. Then so does $\texttt{t}$.
  - Case there exists some term $\texttt{s}_0$ such that $\texttt{t}_0 \longrightarrow \texttt{s}_0$. Then by E-Cast we have $\texttt{(T)t}_0 \longrightarrow \texttt{(T)s}_0$.
- Case T-SCast: Then $\texttt{t}$ has the form $\texttt{(T)t}_0$ and $\bullet \vdash \texttt{t}_0 : \texttt{S}_0$ and $\texttt{S}_0 \not\lhd \texttt{T}$ and $\texttt{T} \not\lhd \texttt{S}_0$ and a *stupid warning* is generated. By induction, there are three subcases.
  - Case $\texttt{t}_0$ is a value. Then $\texttt{t}$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$.
  - Case $\texttt{t}_0$ contains a subexpression of the form $\texttt{(U)(v)}$ where $\bullet \vdash \texttt{v} : \texttt{S}$ and $\texttt{S} \not\lhd \texttt{U}$. Then so does $\texttt{t}$.
  - Case there exists some term $\texttt{s}_0$ such that $\texttt{t}_0 \longrightarrow \texttt{s}_0$. Then by E-Cast we have $\texttt{(T)t}_0 \longrightarrow \texttt{(T)s}_0$.
- Case T-With: Then $\texttt{t}$ has the form $\texttt{t}_0\ \texttt{with X}$ and $\texttt{T}$ has the form $\texttt{U}^{\texttt{X}}$ and $TT(\texttt{X}) = \texttt{expander X of U}_0 \cdots$ and $\Gamma \vdash \texttt{t}_0 : \texttt{U}$ and $\texttt{U} \lhd \texttt{U}_0$. By induction we have three subcases:

- Case $\mathtt{t}_0$ is a value. Then so is $\mathtt{t}$.
- Case $\mathtt{t}_0$ contains a subexpression of the form $\mathtt{(U)(v)}$ where $\bullet \vdash \mathtt{v} : \mathtt{S}$ and $\mathtt{S} \not\lhd \mathtt{U}$. Then so does $\mathtt{t}$.
- Case there exists some term $\mathtt{s}_0$ such that $\mathtt{t}_0 \longrightarrow \mathtt{s}_0$. Then by E-WITH we have $\mathtt{t}_0\ \mathtt{with}\ \mathtt{X} \longrightarrow \mathtt{s}_0\ \mathtt{with}\ \mathtt{X}$.

- Case T-PEEL: Then $\mathtt{t}$ has the form $\mathtt{peel}\ \mathtt{t}_0$ and $\Gamma \vdash \mathtt{t}_0 : \mathtt{T}^{\mathtt{X}}$. By induction we have three subcases:
  - Case $\mathtt{t}_0$ is a value. Then by Lemma 7.14, $\mathtt{t}_0$ has the form $\mathtt{v}\ \mathtt{with}\ \mathtt{X}$. Then by E-PEELWITH we have $\mathtt{peel}\ \mathtt{t}_0 \longrightarrow \mathtt{v}$.
  - Case $\mathtt{t}_0$ contains a subexpression of the form $\mathtt{(U)(v)}$ where $\bullet \vdash \mathtt{v} : \mathtt{S}$ and $\mathtt{S} \not\lhd \mathtt{U}$. Then so does $\mathtt{t}$.
  - Case there exists some term $\mathtt{s}_0$ such that $\mathtt{t}_0 \longrightarrow \mathtt{s}_0$. Then by E-PEEL we have $\mathtt{t}_0\ \mathtt{with}\ \mathtt{X} \longrightarrow \mathtt{s}_0\ \mathtt{with}\ \mathtt{X}$.

References

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