

# On Minimal Energy Skew Routing in Lossy Wireless Sensor Networks

Roozbeh Jafari, Alberto Cerpa, Soheil Ghiasi, Majid Sarrafzadeh

## Abstract

For several sensor networks applications, it is critical to extend the lifetime of each individual sensor node in order to remain operational for the longest time possible. Therefore, the power consumption rate should be evenly distributed over all the nodes in the system. Traditional routing algorithms attempt to minimize the total power consumption of the system, but they do not attempt to evenly distribute the load over all the nodes in the network. In this paper, we present an efficient routing algorithm that minimizes the energy skew among nodes in a network with lossy links. We propose an  $\epsilon$ -optimal polynomial time centralized multi-hop routing technique that maximizes the lifetime a system of distributed power sources, considering the quality of the wireless links and the vagaries of the radio communication channel. Our technique aims to evenly distribute the power consumption rate which yields in a minimal-skew solution. We theoretically prove that our technique is efficient. Finally, we illustrate the quality of the solutions provided by our algorithms on a set of benchmarks that consider the quality of the wireless channels based on models using real RF transceivers. We show that our solution provides significant increase in the lifetime of the network (up to five times) at the cost of a slight increase in the latency of the end-to-end paths (up to 10%).

## Index Terms

Power Optimization, Battery-Aware Routing, Distributed Embedded Systems, Sensor Networks.

## I. INTRODUCTION

Battery-powered networked embedded systems are being widely used in various application domains. Practical limitations do not allow replacement of dead batteries after system deployment. Hence, reduction of energy consumption, as a mechanism to prolong the battery lifetime, has gained significant attention in sensor networks community. Previous research efforts have shown that communication dissipates significant amount of energy [18]. Consequently, design choices and protocols that affect the amount of required transmission have a great impact on system lifetime. For example, minimum-hop routing algorithm reduces the number of transmissions required to deliver a message at the destination, and improves system energy dissipation [9].

On the other hand, many applications heavily rely on robust and non-stop operation of sensing and computing nodes (or simply nodes) to perform their intended functionality. In such classes of applications, failure of a node causes the entire system to fail, and hence, the lifetime of the system is determined by the lifetime of the first node that fails. Consequently, even distribution of energy dissipation becomes an important design concern in addition to total system energy dissipation. Intuitively, even distribution of (minimum-skew) energy dissipation maximizes the lifetime of the nodes with shortest lifetime, and prolongs the application lifetime. The existing energy-aware routing algorithms are generally oblivious to fair distribution of energy consumption among nodes, and exhibit poor results.

In this paper, we present an efficient routing algorithms that minimizes the energy skew among nodes in a mesh network. Our algorithm is a centralized scheme that is globally optimal. This algorithm efficiently finds the best routing strategy for a given traffic pattern in any network topology, including mesh networks. Experimental results on our benchmarks show an increase of up to five times in system lifetime when comparing with optimal shortest path routing with a slight penalty in the overall latency of the end-to-end paths.

Before we continue, we would like to clarify some assumptions used in the remaining of our work. First, our solution is applicable when the quality of the links used for routing does not dramatically change over time. This assumption is confirmed for good quality links by [6]. Second, our solution works only in the case of unicast routing algorithms. The use of broadcast or multicast routing mechanisms has been left for future work.

## II. RELATED WORK

Most of the previous routing protocols ([11][12][13] [3][14]) for wireless *ad-hoc* networks concentrate on finding and maintaining routes in the face of changing topology caused by mobility or other environmental changes. Typical protocols use shortest path algorithms based on hop count, geographic distance, or transmission power. The first two are important in minimizing delay and maximizing throughput. The third objective is peculiar to wireless *ad-hoc* networks, and is important because typically the nodes involved have a limited power supply, and radio communication consumes a large fraction of this supply. To address this issue, several power-aware routing protocols have been developed ([19] [16] [20] [15] [21]). In most of these approaches, the aim is to minimize the energy consumed per packet in order to deliver it to the destination. The typical approach is to use a distributed shortest path algorithm in which the edge costs are related to the power required to transmit a packet between the two nodes involved. The problem with this technique is that nodes on the minimum-energy path are quickly drained of power, affecting the network connectivity when they fail. While some of the most sophisticated routing algorithms associate a cost with routing through a node with low power reserves ([19] [20]), they present at best heuristic solutions.

Researchers have explored the fundamental limits of energy-efficient collaborative data-gathering by deriving upper bounds on the lifetime of increasingly sophisticated sensor networks [2]. But they do not devise any efficient algorithm for routing.

Another method proposed to extend the sensor network operational time consist of organizing the sensors into a maximal number of disjoint set covers that are activated successively. Only the sensors from the current active set are responsible for monitoring all targets and for transmitting the collected data, while nodes from all other sets are in a low-energy sleep mode [4]. The proposed method, however, is a heuristic. Furthermore, a shortest cost path routing algorithm is studied which uses link costs that reflect both the communication energy consumption rates and the residual energy levels at the two end nodes [7]. This approach also formulates the technique as a linear programming problem. Nevertheless, an  $\epsilon$ -optimal polynomial time routing algorithm that maximizes the lifetime of the system with respect to the distributed energy sources has not been devised in any of the prior studies. Furthermore, to the best of our knowledge, no probabilistic distributed routing scheme that attempts to minimize the skew in energy have been proposed.

Authors in [17] present a distributed method for lifetime maximization. Their method starts out with assuming a certain lifetime and attempts to search for the optimal lifetime with a bounded error (by starting out with an arbitrary lifetime) within a certain number of rounds. Our method however, proposes a simpler approach (despite being centralized) that calculates the optimal lifetime in polynomial time. More importantly, the main advantage of our method is that we take into account the lossy channels. Authors in [10] also propose a similar approach assuming the communication links are lossless.

## III. PRELIMINARIES AND MODELS

The problem at hand is to design an effective routing mechanism to maximize the application lifetime, which is given by the lifetime of the first node that fails. If nodes have identical initial energy resources, the objective can be rephrased as minimizing the maximum energy consumption rate of the nodes.

We model the network using an unconstrained connected directed graph  $G_s = (V_s, E_s)$ , where  $V_s$  denotes set of nodes  $v_1, v_2, \dots, v_n$ , and  $E_s$  represents the set of directed links  $l_{ij}$ . Note that directed graphs can be utilized to model any network topology. We will show that our algorithm is optimal for any directed graph.

We represent the energy consumed by node  $v_i$  to transmit or receive an information unit (also called packet or message in this paper) by  $t_i$  and  $r_i$ , respectively. These two parameters account for the energy consumption incurred by communication and computation (or possibly other sources of energy consumption) required to transmit or receive a packet. We assume that node  $v_i$  has an energy level  $E_i$ , which is initially equal to  $E_{i,0}$  ( $\forall(i, j) \in V_s E_{i,0} > 0$ ). Note that  $E_i$  decreases as packets are received and/or sent by node  $v_i$ .

Let  $f_{ij}$  be the number of packets transmitted from node  $i$  to its immediate neighbor node  $j$  over link  $l_{ij}$  in a routing scenario. We assume that network has some periodicity property in data transmission, in which case,  $f_{ij}$  can be interpreted as the number of packets transmitted over  $l_{ij}$  in unit of time or its *transmission rate*. The lifetime of node  $v_i$  is defined as follows:

$$T_i = \frac{E_{i,0}}{(t_i \cdot \sum_{\forall j; l_{ij} \in E_s} f_{ij}) + (r_i \cdot \sum_{\forall j; l_{ji} \in E_s} f_{ji})} \quad (1)$$

Where  $\sum_{\forall j; l_{ij} \in E_s} f_{ij}$  is the total number of packets transmitted from node  $v_i$ , and similarly,  $\sum_{\forall j; l_{ji} \in E_s} f_{ji}$  is the total number of packets received by  $v_i$ .

#### IV. DEFINITION

In the following formulation we attempt to minimize the skew in energy consumption (due to wireless communication) in the network. The skew is defined as follows:

There exists exponential number of paths connecting source to destination nodes. For every path, we form the following definition. There exists a node in every path that has the maximum energy consumption (ideally if each path is isolated from the rest of the network, the energy consumption rate must be identical throughout the path, however, in reality, nodes may have incoming/outgoing edges from/to the rest of the network. Therefore, the energy consumption rate may not be necessarily uniformly distributed). In every path  $P_i$  connecting a source to a destination node, we identify the node with maximum energy consumption rate as  $p_{imax}$ . We define the skew of energy consumption as the difference between  $p_{imax}$  and  $p_{jmax}$  of paths  $i$  and  $j$ . In the following Sections, we will illustrate how our formulation minimizes the difference of maximum energy consumption in every two paths. Please note that despite the number of paths are exponential, the upper bound on the number of nodes with maximum energy consumption is still  $n$  (where  $n$  is total number of nodes). The definition of minimum skew is rephrased in Equation 2.

$$\begin{aligned} &\forall P_i \& P_j \text{ connecting source(s) to destination(s)} \\ &\text{Minimize } |p_{imax} - p_{jmax}| \end{aligned} \quad (2)$$

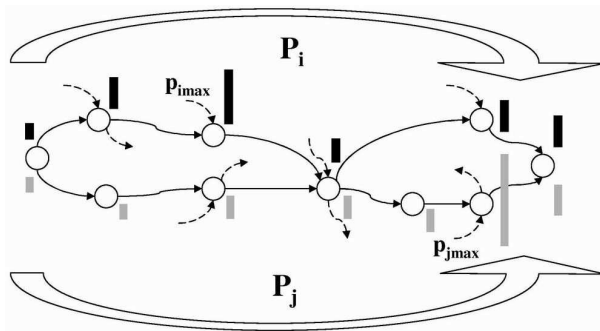


Fig. 1. Minimum Skew Definition

#### V. MINIMAL-SKEW ROUTING

##### A. Problem Formulation and $\epsilon$ -Approximation Algorithm

Given a sensor network  $G_s$  and a specific traffic pattern, the objective is to route the packets so that after completion of packet routing, the minimum remaining battery energy among all of the nodes is maximized. This objective would maximize the lifetime of the first failing node, and subsequently, maximizes the application lifetime. We assume that there is a specific node  $t$  in  $G_s$ , which serves as the gateway or base station or destination node, and all of transmitted packets have to be delivered to  $t$ . Specifically, the given traffic pattern is composed of a set of  $(node, quantity)$  pairs, which specifies the source and the number of packets that need to be sent to  $t$ . Our formulation, however, accommodates a more extensive objective which covers the aforementioned objective as described in Equation 2.

Furthermore, we temporarily assume that all nodes have identical initial energy levels. In Subsection V-C, we will extend our results to show that our model and technique is extensible to cases where nodes have dissimilar initial energy levels.

Each packet transmitted from a source node to  $t$  can be viewed as a unit flow in the network  $G_s$ . More precisely,  $x_{ij}$  units of flow going through the link  $l_{ij}$  represent  $x_{ij}$  packets transmitted from  $v_i$  to  $v_j$ . This traffic decreases the energy resources at  $v_i$  and  $v_j$  by  $x_{ij} \times t_i$  and  $x_{ij} \times r_j$ , respectively. The given traffic pattern can be interpreted as a flow supply vector,  $FS$ , which specified the amount of flow supply at nodes. All of packets have to be received at node  $t$ , and hence, Node  $t$  is the only node with negative flow supply (positive flow demand). The supply for node  $t$  is set such that the total supply over all the nodes in the network is zero.

The problem of packet routing is equivalent to finding a feasible network flow in  $G_s$ . Intuitively, the objective of maximizing the minimum remaining energy of the nodes is equivalent to minimizing the maximum flow passing through nodes. Note that the remaining energy at intermediate (not a source or destination) node  $v_i$  with  $x_i$  units of flow passing through it is  $E_i - x_i \cdot (r_i + t_i)$ . In the remainder of this section, we transform  $G_s$  to a new network  $G_t$  in which, min-cost flow solution generates the optimal solution. We proceed to describe the transformation procedure, along with mathematical properties of our technique and proofs of correctness.

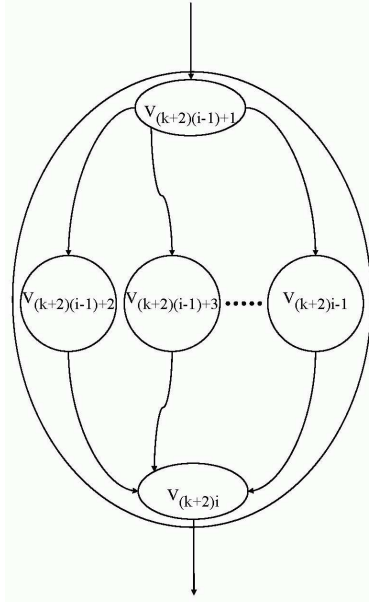


Fig. 2. Node partitioning

We construct the network  $G_t = (V_t, E_t)$  from the graph  $G_s$  according to the following rules. Each node in graph  $G_s$  is split into  $k + 2$  nodes where  $k$  is a tunable parameter that controls the accuracy of the solution. We refer to the resulting set of nodes as a *partition*. In each partition, two nodes serve as receiver and transmitter (or input and output) and the rest of the nodes are called splits. Figure 2 illustrates an example partition in which, nodes  $v_{(k+2)(i-1)+1}$  and  $v_{(k+2)i}$  are the receiver and the transmitter of the partition, respectively. Figure 3 shows an example network  $G_s$  and the resulting network  $G_t$  after transformation.

Each intermediate node  $v_i$  in  $G_s$  consumes  $r_i + t_i$  units of energy to relay a packet. Therefore,  $v_i$  can relay at most  $u_i = E_i / (r_i + t_i)$  packets before running out of battery.  $u_i$  forms an upper bound on the number of packets that can pass through node  $v_i$ . In order to embed this constraints into the problem, we assign the flow upper bound of  $u'_i = u_i / k$  to each of the split nodes. The upper bounds guarantee that a partition cannot pass more flow after relaying  $u_i$  units of flow, or equivalently, a node cannot route packets after its battery is dead. The flow upper bound for all of the receiver and transmitter nodes is infinity. Note that the upper bound  $u'_i = u_i / k$  on the split nodes in partition  $i$ , implicitly sets the upper bound of  $u_i$  for the receiver and transmitter nodes of the partition. The receiver, transmitter and split nodes are assigned a special sequence of costs for passing the unit flow. The cost associated with receiver and transmitter nodes of a partition is zero. The costs associated with split nodes of a partition are increasing from left to right (Figure 2). This directs the min-cost flow solution to utilize the split nodes from left to right, when passing flow through the partition. Specifically, we assign the following costs to the nodes in partition  $i$ :

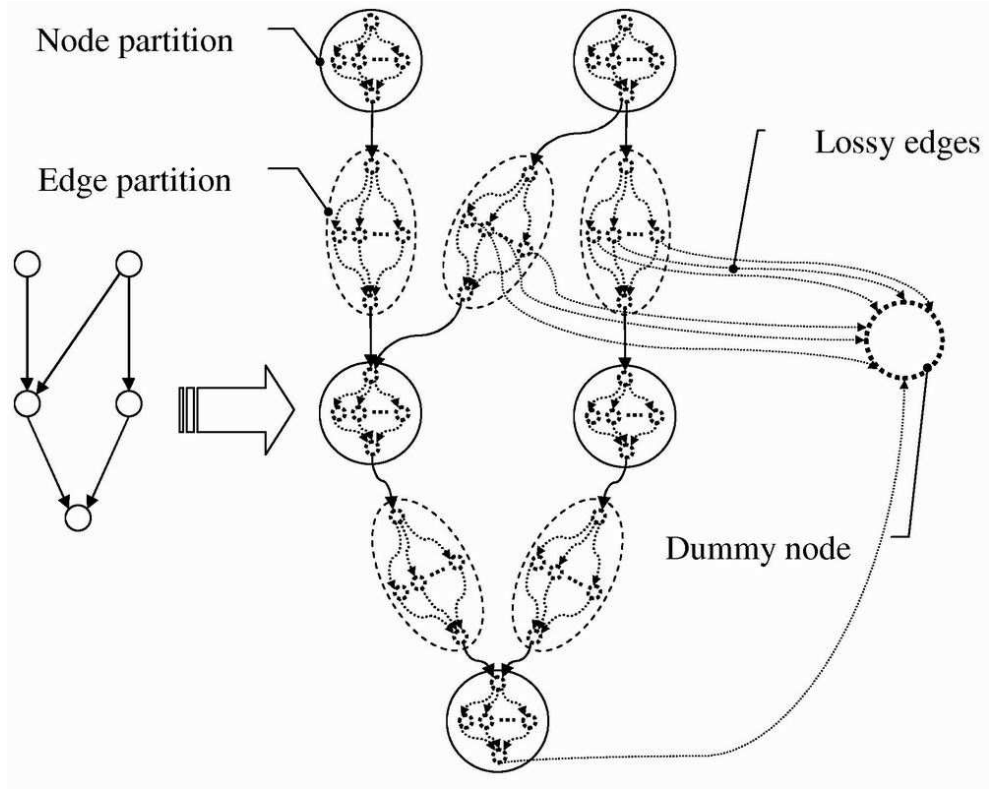


Fig. 3. Original network and the resulting network after partitioning

$$\begin{aligned}
 c_{(k+2)(i-1)+1} &= 0 \\
 c_{(k+2)(i-1)+2} &= 1 \\
 c_{(k+2)(i-1)+3} &= n + \varepsilon \\
 c_{(k+2)(i-1)+4} &= n(n + 1 + \varepsilon) + \varepsilon \\
 &\vdots \\
 c_{(k+2)i-1} &= n \left( \sum_{j=2}^k c_{(k+2)(i-1)+j} \right) + \varepsilon \\
 c_{(k+2)i} &= 0
 \end{aligned} \tag{3}$$

The cost on each split node is enforced such that it would be greater than the cumulative cost of the split nodes with smaller indices over all of the partitions. Intuitively, our cost assignment technique enforces the optimal min-cost flow solution to utilize the split nodes with smaller indices, before trying to utilize a particular split node. This simple yet effective idea exhibits the main property of our technique by which, we minimize the maximum energy consumption of the nodes. For simplicity, we define cost rank where cost rank  $R_i$  has the following property:

$$R_i > \sum_{j < i} n \cdot R_j \tag{4}$$

Consequently, Equations 3 may be reworded as follows:

$$\begin{aligned}
 c_{(k+2)(i-1)+1} &= 0 \\
 c_{(k+2)(i-1)+2} &= R_1 \\
 c_{(k+2)(i-1)+3} &= R_2 \\
 c_{(k+2)(i-1)+4} &= R_3 \\
 &\vdots \\
 c_{(k+2)i-1} &= R_k \\
 c_{(k+2)i} &= 0
 \end{aligned} \tag{5}$$

The loss in edges may also be accommodated easily. For the sake of simplicity, we add the extensions for lossy edges in Section V-B. The aforementioned procedure provides a well-defined set of steps to create  $G_t$  from  $G_s$ . A packet routed from a sensor node  $s$ , to the base station  $t$ , in  $G_s$ , corresponds to a unit of  $s \rightarrow t$  flow in  $G_t$ . Similarly, our partition construction scheme along with the upper bound settings on split nodes guarantee that any flow solution in  $G_t$  represents a feasible routing solution in  $G_s$ . Note that the given traffic pattern for  $G_s$  can be readily translated into a flow supply vector ( $FS$ ) for  $G_t$ .  $FS$  specifies the amount of flow supply or demand for all of the nodes.

We now prove that our cost assignment strategy implies that the min-cost flow solution in  $G_t$  corresponds to a routing scheme in  $G_s$  that minimizes the maximum consumed energy at the nodes (or equivalently, maximizes the minimum remaining energy over all of the nodes). Let  $x_{ij}$  represent the amount of flow on edge  $(i, j)$ . Let  $y_i$  represent the amount of flow going through split node  $k$  in partition  $i$ . Similarly, let  $c_i$  denote the associated cost of unit flow passing through that node in  $G_t$ . The min-cost flow problem for graph  $G_t$  with the given supply and demand vector,  $FS$ , can be written as:

$$\text{Minimize } \sum_{\forall i} c_{i,k} y_{i,k} \quad (6)$$

Subject to:

$$\sum_{\{j:(i,j) \in A\}} x_{ij} - \sum_{\{j:(j,i) \in A\}} x_{ji} = b(i) \quad (7)$$

$$0 \leq x_{ij} \leq u_{ij}; \quad \forall (i, j) \in V_t \quad (8)$$

$$0 \leq y_i \leq u'_i; \quad \forall (i) \in V_t \quad (9)$$

Equation (7) is the flow conservation condition at each node and Equations (8) and (9) are capacity constraints for the arcs and the nodes respectively. We assume that the lower bounds on arc flows as well as the lower bounds on node flows are zero. Moreover, the number of nodes in network  $G_s$  is denoted by  $n$ .

The flow that passes through the receiver or transmitter nodes of a partition represents the total flow passing through that partition, or equivalently, it determines the energy consumption at the sensor node corresponding to the partition. The more the flow, the shorter the lifetime of the sensor node. Therefore, the objective is to minimize the maximum amount of flow passing through partitions.

The flow passing through any partition has to pass through its split nodes, and subsequently, min-cost flow solutions utilize the splits with lower costs before higher cost splits. We denote the number of split nodes in partition  $i$  that carry a non-zero flow by  $\psi_i$ .

$$\psi_i = \left\lceil \frac{\sum_{j=2}^{k+1} y_{(k+2)(i-1)+j}}{u'_i} \right\rceil \quad (10)$$

*Theorem 5.1:* The objective function in Equation 6 minimizes the maximum flow in the nodes of network  $G_s = (V_s, E_s)$  with maximum error  $\varepsilon$  where  $\varepsilon \leq u'_i = u_i/k = 1/k$ . This is equivalent to maximizing the minimum battery lifetime of the nodes in network  $G_s$  (or its transformed network  $G_t$ ) and therefore analogous to maximizing the lifetime of the system.

*Proof:* Proof is formed by contradiction. Assume our technique generates solution  $L$  where the flow entering each partition is represented by  $f_i$ . Let  $f_{\max} = (\max(f_i) \forall i)$ . Assume there exist another solution  $L^*$  with maximum flow denoted by  $f_{\max}^*$  where  $f_{\max}^* < f_{\max} + \varepsilon$ .

Thus, (10) conveys that:

$$\psi_{\max}^* < \psi_{\max}$$

Considering the cost on splits in Figure 2 and Equation (3), we conclude that the overall cost of flow for solution  $L$  is greater than the overall cost of flow for solution  $L^*$ . This contradicts the optimality of min-cost flow technique since the solution found ( $L$ ) does not have the minimum cost. Therefore, by contradiction, the solution  $L^*$  cannot exist and our formulation minimizes the maximum flow (or maximizes the minimum lifetime).

Due to the flow conservation condition in min-cost flow technique, it is trivial that the flow in any nodes may not be reduced individually to minimize the objective function.

The intuition behind our proposed technique is that the cost assignment on the splits forces the network to route a flow from the  $k$ th split of node  $v_i$ , if it cannot be routed through any number of other nodes whose  $(k - 1)$ th splits is empty. ■

*Theorem 5.2:* The solution  $L$ , generated by our technique, minimizes the difference of the maximum node flows throughout every two disjoint paths connecting source to a destination node (with tolerance of  $\varepsilon = 1/k$  - minimal-skew). In other words, these two nodes further must have the minimum lifetime of the nodes along each path and are regarded as the bottleneck nodes (i.e. such nodes have the maximum energy consumption rate along their corresponding paths).

*Proof:* This proof is presented by contradiction as well. Assume there exists a feasible solution  $L^*$  which was transformed from  $L$  and the flows of two partitions  $i$  and  $j$  was altered such that their difference is reduced. We denote the new flows in solution  $L$  by  $f_i$  and  $f_j$ . Without loss of generality, we assume  $f_i < f_j$ .

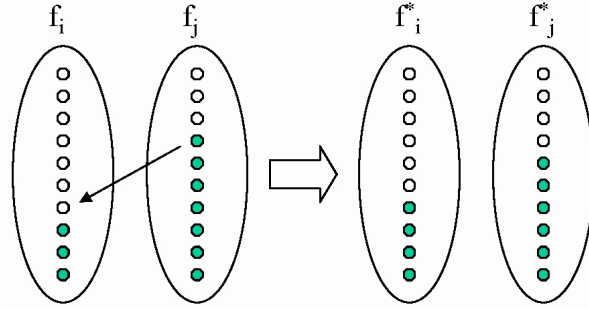


Fig. 4. Flow exchange to obtain minimal-skew solution

We investigate both possible scenarios. One scenario occurs when the transformation involves changing the flow in only one partition (increasing  $f_i$  or decreasing  $f_j$ ). According to the flow conservation theorem, this scenario is not feasible. The other scenario, as shown in Figure 4, takes place when the flow of partition  $i$  is increased and the flow of partition  $j$  is decreased (by a value of greater than  $\varepsilon$ ) such that:

$$\begin{aligned} |f_j^* - f_i^*| &< |f_j - f_i| + \epsilon \\ \text{and} \\ f_j^* + f_i^* &= f_j + f_i + \epsilon \end{aligned} \quad (11)$$

From Equation 9, it can be easily shown that:

$$f_i < f_i^* < f_j^* < f_j \quad (12)$$

Therefore, the number of splits that are utilized for passing the flow follows the same convention:

$$\psi_i < \psi_i^* < \psi_j^* < \psi_j \quad (13)$$

As discussed before, in each partition, each split has a cost that alone is greater than “ $n$ ” times the cumulative cost of all the precedent splits with smaller indices ( $n =$  number of nodes in  $G_s$  (original graph)). Since the solutions  $L$  and  $L^*$  are similar except in partitions  $i$  and  $j$ , therefore, the overall cost of solution  $L$  is greater than the cost of  $L^*$  ( $\psi_j > \psi_j^*$ ). This contradicts the optimality of min-cost flow algorithm as the solution  $L$  must have the minimum cost. Hence, solution  $L$  cannot exist. ■

*Theorem 5.3:* The solution of minimal-skew routing is unique in the sense that the lifetime of nodes in the network in descending order is unique.

*Proof:* Assume  $F$  and  $F'$  are the vectors containing the flow of all nodes in descending order for two optimal solutions,  $L$  and  $L'$ . Obviously  $F[1] = F'[1]$ , otherwise the two total costs would be different (that contradicts the optimality of the solution). Inductively, this argument holds for every index. Assume  $F[i] = F'[i]$ , for  $i = 1 \dots k$ . If  $F[k+1] < F'[k+1]$  then because of the special cost assignment of the splits, the cost of  $F'[k+1]$  itself would be larger than the total cost of  $F[i]$ s,  $i = k+1 \dots n$ . Therefore, the total cost of solution  $L'$  is greater than the cost of  $L$ . This contradicts the optimality of solution  $L'$ . This completes the proof. ■

### B. Discussion on Lossy Channels

In general, accommodating lossy communication channels in minimum cost flow technique is not quite an easy task due to flow conservation conditions. In the case where the existence of lossy communication links are inevitable, the packets received at the receiver is less than the number of packets transmitted. This scenario corresponds to flow attenuation in our minimum cost formulation. To address the data/flow loss in communication links, ideally we prefer to remove certain portion of the flow traveling through an edge and transfer it to a auxiliary node which is considered as a "dummy" sink node. Other gateways and sink nodes are also connected to the aforementioned node. This can be accomplished by adding intermediate nodes on every interconnection edge. Moreover, the intermediate nodes have to be connected through "lossy" edges with cost zero a "dummy" sink node. The upper bound on "lossy" edges corresponds to the loss rate of every interconnection edge. The limitation we face, however, is that the initial flow in every edge tends to be directed through the lossy edge until the "lossy" edge reaches its capacity. This causes a potential problem where the interconnection edges are not utilized with their full capacity. Therefore, this will incur in an increase in loss rate. To address this issue, we propose the following transformation:

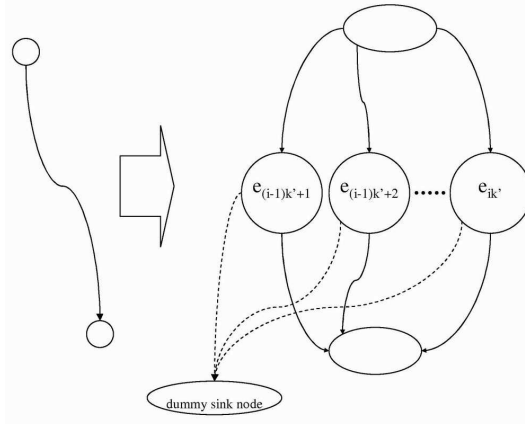


Fig. 5. Edge splitting

Each edge is divided into  $k$  split edges  $e_{ik}, \dots, e_{i(k+1)-1}$  and  $k$  lossy edges  $l_{ik}, \dots, l_{i(k+1)-1}$  connecting intermediate nodes to a dummy node as shown in Figure 5. The costs on split edges correspond to  $R_1, R_3, \dots, R_{2k}$  while the cost on lossy edges correspond to  $R_2, R_4, \dots, R_{2k+1}$ . This will ensure that initially edge  $e_{ik}$  is utilized and subsequently lossy edge  $l_{ik}$ . Despite, we attempt to characterize the probabilistic nature of loss with a deterministic approach, over a long run, the behavior of our model become analogous to steady behavior of real systems. The above cost distribution will ensure that edges  $e_{ik}, l_{ik}, e_{ik+1}, \dots, e_{i(k+1)-1}, l_{i(k+1)-1}$  are utilized accordingly. Changing the cost ranks associated with the edges enables us to accommodate other combinations of edge utilization. All Theorems and Lemmas proven in the this Section still holds for graphs with lossy edges (with minor modifications). The proofs are omitted due to lack of space.

### C. Dissimilar Initial Energy Levels

Throughout our formulation, we assumed that the initial energy levels in all nodes are similar. Dissimilar initial energy levels, however, can be simply accommodated by modifying the capacity of the splits (upper-bounds on the splits). Throughout our formulation, we assumed that the initial energy levels in all nodes are similar. Dissimilar initial energy levels, however, can be simply accommodated by modifying the capacity of the splits (upper-bounds on the splits). The main property of our technique is that we assign various costs to different levels of energy stored in a battery. When a battery is fully charged, it can be used more easily than the case it has half of the full charge. Therefore, in the case where some nodes in the network do not have full energy level, the already used portion of their battery corresponds to the splits with lesser cost. Hence, those splits could be assigned upper bound of zero in the min-cost flow formulation as if they have been already used.



#### D. Time Complexity Analysis

The formulation used for the proposed min-cost flow problem in Equations (2) through (9) is an LP formulation which can be solved with standard LP solvers. Throughout our experiments, we used Matlab as an LP solver. The use of LP-solvers enabled us the ability to have non-integral capacity in the formulation and non-integral flow in the solution. In order to provide combinatorial algorithms to solve such problem, integrality constraints must be enforced. Fortunately all the problem parameters such as supply/demands and capacities assigned to splits can be scaled by a factor of  $k$  (the number of splits) and meet the integrality constraints. Therefore, the time complexity of our technique is after scaling would be  $O((m \log(nk))(m + nk \log(nk)))$  where  $m$  is the number of edges in the graph;  $n$  and  $k$  are the number of nodes and splits in each partition, respectively. The time complexity can be easily derived from the time complexity of the min-cost flow algorithms and the size of our constructed network ( $O(nk)$ ). The original time-complexity of min-cost flow is reported in [1].

#### E. Discussion on Multicommodity

In scenarios where several sources and destinations are involved and the communication pairs exchange “different” type of messages, routing problems can be modeled as multicommodity flow. Our technique can not directly address this class of problems [8]. The problem itself is known to be NP-complete. However, we believe that our methodology can be applied in conjunction with the known heuristics for multicommodity flow problems and generate reasonable results. We have not considered this class of networks but we plan to study it in near future.

### VI. EXPERIMENTAL RESULTS

We generated various benchmarks based on random graphs with relatively large number of nodes to illustrate the effectiveness of our centralized technique. For simplicity, we assumed that the energy level in all benchmarks is uniform. Our benchmark sets resemble real-world networks. In most network applications, it might be unlikely that a large portion of sensor nodes are placed within close proximity of each other. Instead, they are placed on a grid with certain random properties. It can be envisioned as a ‘locally random globally regular networks.’ Such scenario can be imagined with the following example: Certain number of sensor nodes is required to be placed in a building. Each room has a specific number of sensor nodes which is constant, yet, the position of the nodes is random within each room. We call this set as random networks with grid distribution. We generate such benchmarks by dividing the area into unit-size tiles. A tile that does not have a sensor node is selected randomly. We place a node in the tile with uniform distribution. This procedure is repeated until all tiles are covered. If more sensor nodes are required to be inserted, the same course is recurred until all sensor nodes are placed. One hundred sensors are placed within areas of size  $160 \times 40$ ,  $160 \times 60$ ,  $160 \times 80$ ,  $200 \times 40$ ,  $200 \times 60$ ,  $200 \times 80$  and  $200 \times 100$ . We refer to this set of benchmarks as “grid” networks. In all networks, three source nodes are placed on far left side of the square area while the destination nodes/ gateways are placed on the right side of the square. This particularly assist us to place the source/destination not within the close proximity of each other. The connectivity between nodes is determined by statistical models developed using real RF transceivers under different conditions and scenarios [5]. Only communication links with reliability greater than 80% are considered. In the next set of experiments, the locations of the nodes are generated conforming to a random uniform distribution over areas various size. We refer to this class of random networks as “random” networks.

Firstly, we illustrated the effectiveness of our technique on two benchmarks. One benchmark belongs to our class of “grid” networks while the other resembles “random” network. Each network consists of 100 nodes scattered over an area of 200 square meters.

The graph topology of “grid” and “random” networks are illustrated in Figures 6 and 8 respectively.

Figure 7 depict the normalized energy consumption rate of the nodes in the system sorted in ascending order for various numbers of splits ( $k$ ) in the “grid” network. The effectiveness of our algorithm on how the energy consumption rate becomes evenly distributed is clearly demonstrated when  $k$  is increased. Figure 7 illustrates the same graph for the “random” network shown in Figure 8.

In the next set of experiments, the size on benchmarks is varied. For every particular size, twenty benchmarks are generated with three source and three destination nodes. To compare our scheme against other routing algorithms, we consider a shortest path routing algorithm based on minimum cost flow. Figures 10, 11 , 12 and 13 demonstrate the lifetime resulted from our scheme compared to min-cost technique. In all diagrams, each data-point corresponds to

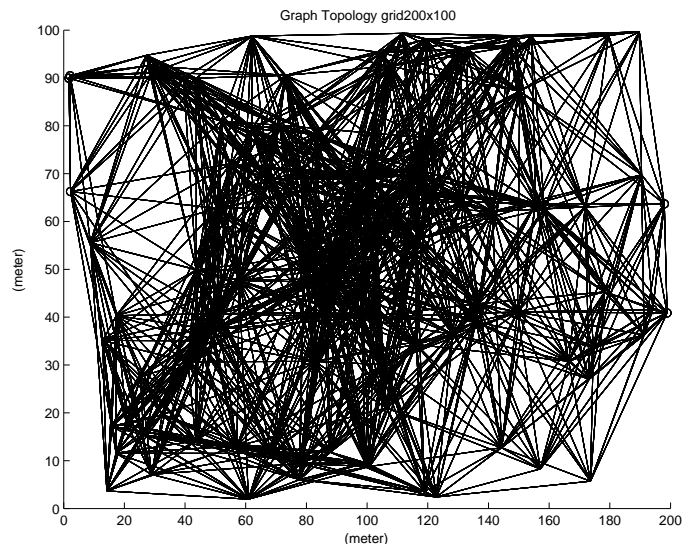


Fig. 6. Graph topology ( $n=100$ , grid distribution)

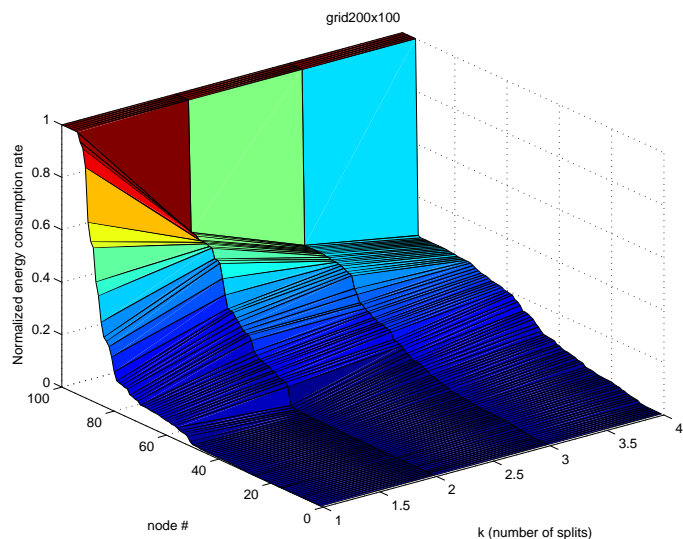


Fig. 7. Normalized energy consumption rate in sorted order ( $n=100$ , grid distribution)

the average taken over twenty benchmarks. Throughout all diagrams, as the number of splits increases, the lifetime improves due to forcing the network to utilize vertex disjoint paths more effectively. Overall, the average lifetime of the systems (for  $k = 4$ ) is increased by a factor of 4.38 compared to min-cost routing approach.

Figures 14, 15, 16 and 17 exhibits the delay trade-offs of our routing algorithm with respect to the min-cost shortest path for various benchmarks.

Overall, the average delay of our scheme (for  $k = 4$ ) is 10% greater than the min-cost shortest path. The average delay is not vastly increased due to existence of multiple vertex disjoint paths between sources and destinations in our benchmarks. The average delay also increases slightly as the  $k$  (number of splits) is increased.

In general, highly connected networks such as grids provide a large number of parallel paths between nodes which is of our interest and enhances the flexibility of data routing.

## VII. CONCLUSION

We proposed a polynomial time  $\varepsilon$ -optimal technique for multi-hop routing in wireless networks with distributed battery sources. Our technique maximizes the lifetime of the system. Furthermore, it evenly distributes the energy consumption rate which yields in a minimal-skew solution for node utilization. We theoretically proved that our

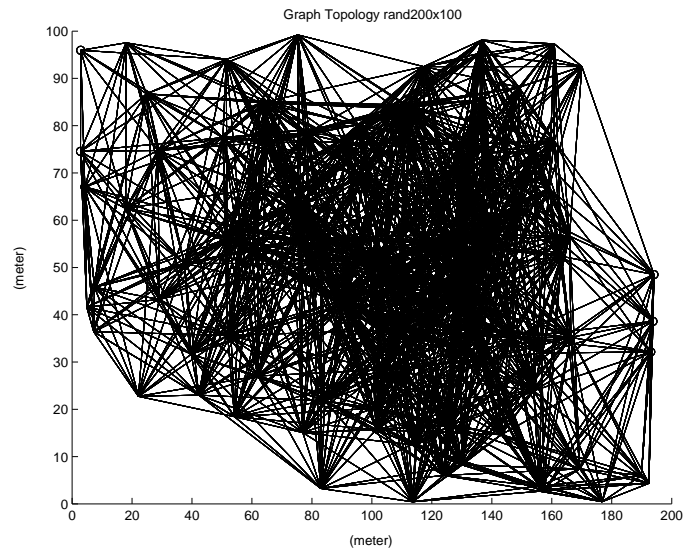


Fig. 8. Graph topology ( $n=100$ , random distribution)

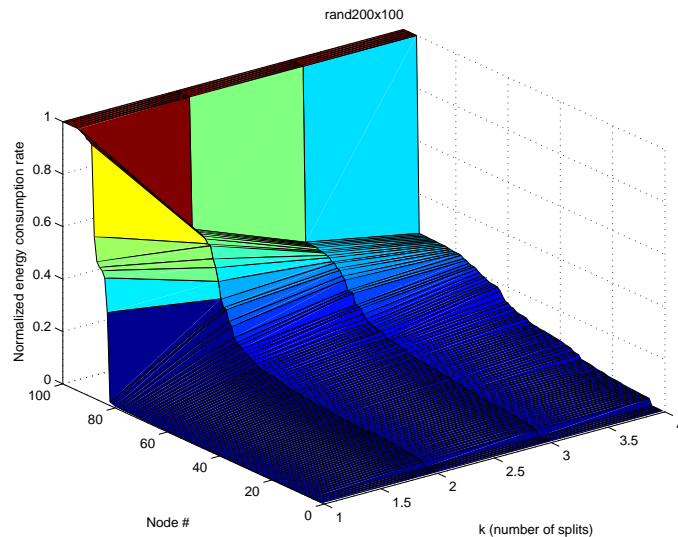


Fig. 9. Normalized energy consumption rate in sorted order ( $n=100$ , random distribution)

technique is efficient and has polynomial time complexity. Furthermore, our technique accommodates routing through lossy links while the optimality is not sacrificed. Our investigation on various benchmarks revealed the quality of the solutions generated by our methodology even with a small number of splits.

## REFERENCES

- [1] Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin. *Network flows: theory, algorithms, and applications*. Prentice-Hall, Inc., 1993.
- [2] M. Bhardwaj and A. Chandrakasan. Bounding the lifetime of sensor networks via optimal role assignments, 2002.
- [3] Prosenjit Bose, Pat Morin, Ivan Stojmenovi&#263;, and Jorge Urrutia. Routing with guaranteed delivery in ad hoc wireless networks. *Wirel. Netw.*, 7(6):609–616, 2001.
- [4] M. Cardei and D.-Z. Du. Improving wireless sensor network lifetime through power aware organization. *ACM Wireless Networks*, 11(3), May 2005.
- [5] Alberto Cerpa, Jennifer L. Wong, Loane Kuang, Miodrag Potkonjak, and Deborah Estrin. Statistical model of lossy links in wireless sensor networks. In *Proceedings of the ACM/IEEE Fourth International Conference on Information Processing in Sensor Networks (IPSN'05)*, Los Angeles, CA, USA, April 25–27 2005. ACM/IEEE.
- [6] Alberto Cerpa, Jennifer L. Wong, Loane Kuang, Miodrag Potkonjak, and Deborah Estrin. Temporal properties of low power wireless links: Modeling and implications on multi-hop routing. In *Proceedings of the Sixth ACM/IEEE MOBIHOC'05*, Urbana-Champaign, IL, USA, May 25–28 2005. ACM/IEEE.

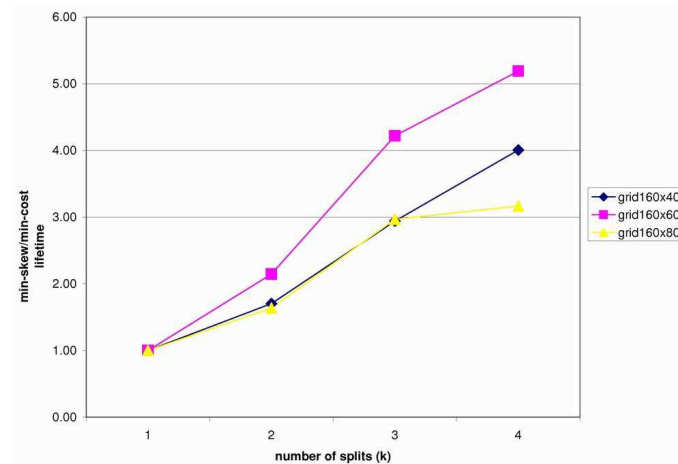


Fig. 10. min-skew/min-cost lifetime for various number of splits - k (n=100, grid distribution)

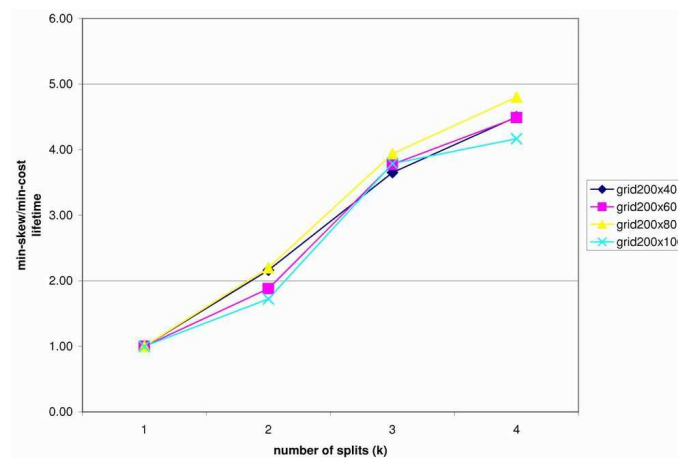


Fig. 11. min-skew/min-cost lifetime for various number of splits - k (n=100, grid distribution)

- [7] Jae-Hwan Chang and Leandros Tassiulas. Maximum lifetime routing in wireless sensor networks. *IEEE/ACM Trans. Netw.*, 12(4):609–619, 2004.
- [8] M. R. Garey and D. S. Johnson. *Computers and intractability: a guide to the theory of NP-completeness*. W. H. Freeman, 1979.
- [9] Chalermek Intanagonwiwat, Ramesh Govindan, Deborah Estrin, John Heidemann, and Fabio Silva. Directed diffusion for wireless sensor networking. *IEEE/ACM Trans. Netw.*, 11(1):2–16, 2003.
- [10] Roozbeh Jafari, Foad Dabiri, and Majid Sarrafzadeh.  $\epsilon$ -optimal minimal-skew battery lifetime routing in distributed embedded systems. *Journal of Low Power Electronics*, 1(2):97–107, 2005.
- [11] David B Johnson and David A Maltz. Dynamic source routing in ad hoc wireless networks. In Imielinski and Korth, editors, *Mobile Computing*, volume 353. Kluwer Academic Publishers, 1996.
- [12] Shree Murthy and J. J. Garcia-Luna-Aceves. An efficient routing protocol for wireless networks. *Mob. Netw. Appl.*, 1(2):183–197, 1996.
- [13] Vincent D. Park and M. Scott Corson. A highly adaptive distributed routing algorithm for mobile wireless networks. In *INFOCOM '97: Proceedings of the INFOCOM '97. Sixteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Driving the Information Revolution*, page 1405, Washington, DC, USA, 1997. IEEE Computer Society.
- [14] Charles E. Perkins and Elizabeth M. Royer. Ad-hoc on-demand distance vector routing. In *WMCSA '99: Proceedings of the Second IEEE Workshop on Mobile Computer Systems and Applications*, page 90, Washington, DC, USA, 1999. IEEE Computer Society.
- [15] Ram Ramanathan and Regina Hain. Topology control of multihop wireless networks using transmit power adjustment. In *INFOCOM (2)*, pages 404–413, 2000.
- [16] V. Rodoplu and T. Meng. Minimum energy mobile wireless networks, 1998.
- [17] Arvind Sankar and Zhen Liu. Maximum lifetime routing in wireless ad-hoc networks. In *INFOCOM*, 2004.
- [18] Victor Shnayder, Mark Hempstead, Bor rong Chen, Geoff Werner Allen, and Matt Welsh. Simulating the power consumption of large-scale sensor network applications. In *SensSys '04: Proceedings of the 2nd international conference on Embedded networked sensor systems*, pages 188–200, New York, NY, USA, 2004. ACM Press.
- [19] Suresh Singh, Mike Woo, and C. S. Raghavendra. Power-aware routing in mobile ad hoc networks. In *MobiCom '98: Proceedings of*

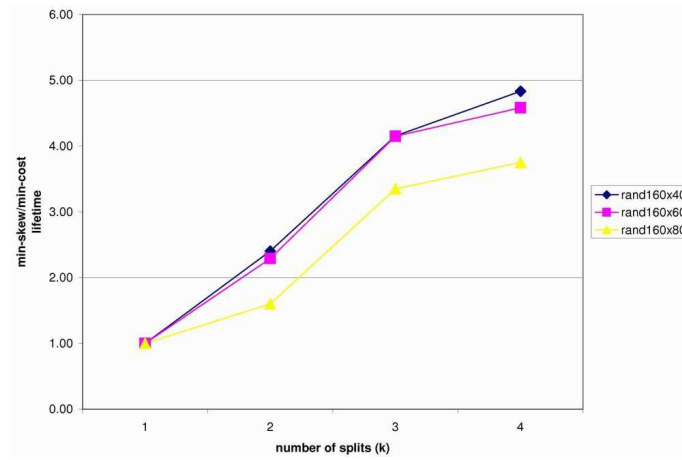


Fig. 12. min-skew/min-cost lifetime for various number of splits - k (n=100, rand distribution)

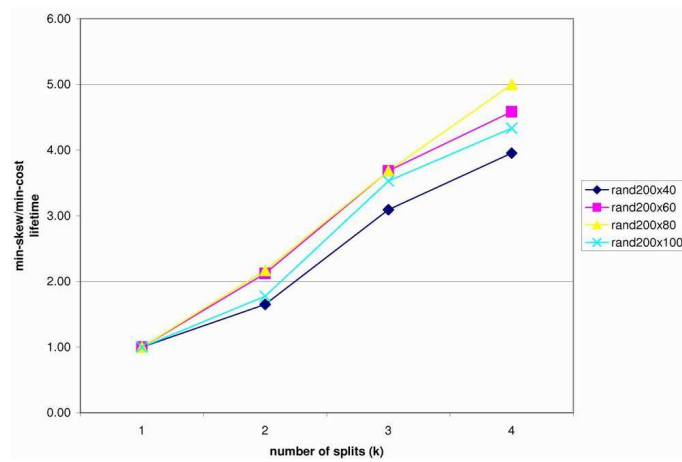


Fig. 13. min-skew/min-cost lifetime for various number of splits - k (n=100, rand distribution)

*the 4th annual ACM/IEEE international conference on Mobile computing and networking*, pages 181–190, New York, NY, USA, 1998. ACM Press.

- [20] Ivan Stojmenovic and Xu Lin. Power-aware localized routing in wireless networks. *IEEE Transactions on Parallel and Distributed Systems*, 12(11):1122–1133, 2001.
- [21] Roger Wattenhofer, Li Li, Paramvir Bahl, and Yi-Min Wang. Distributed topology control for wireless multihop ad-hoc networks. In *INFOCOM*, pages 1388–1397, 2001.

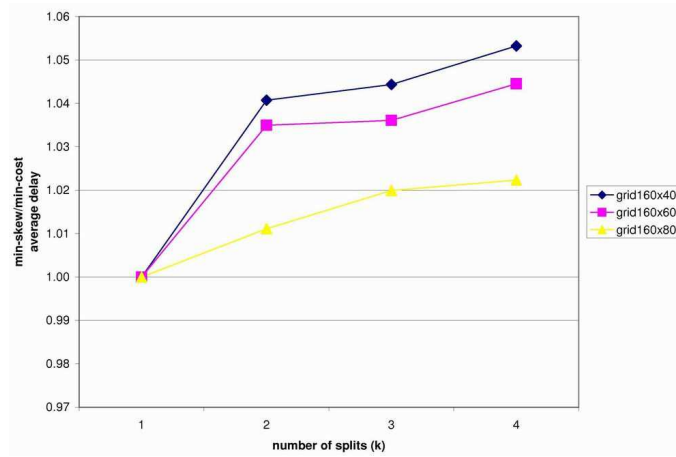


Fig. 14. Average delay of min-skew/min-cost for various number of splits - k (n=100, grid distribution)

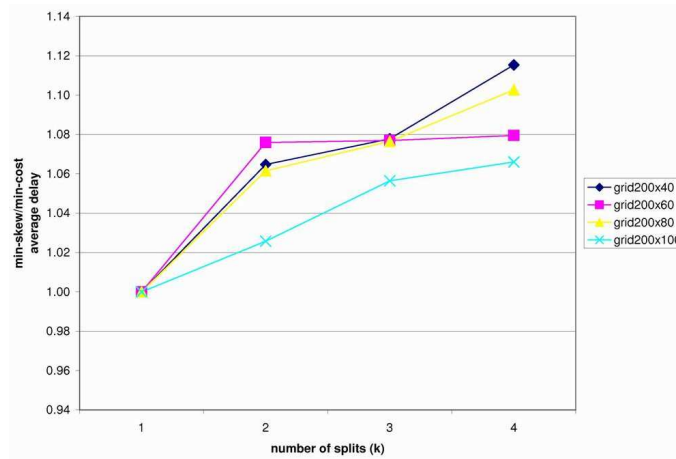


Fig. 15. Average delay of min-skew/min-cost for various number of splits - k (n=100, grid distribution)

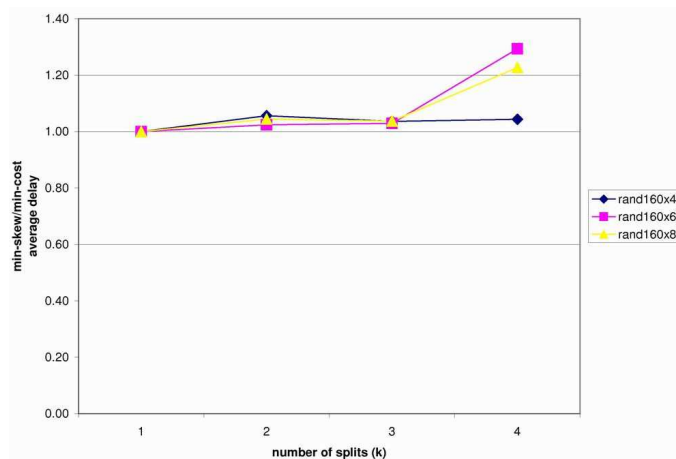


Fig. 16. Average delay of min-skew/min-cost for various number of splits - k (n=100, rand distribution)

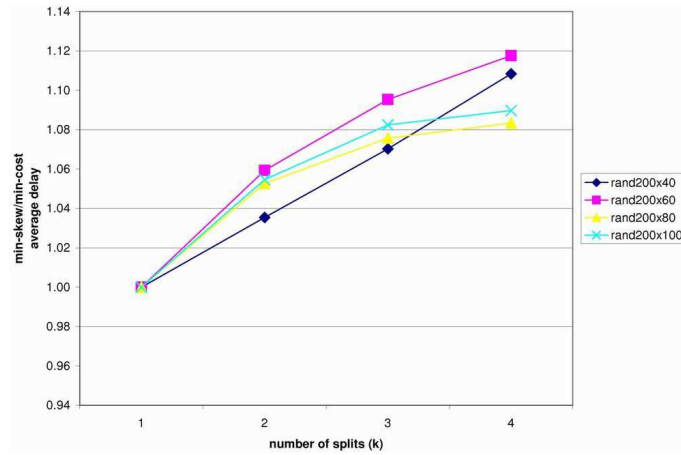


Fig. 17. Average delay of min-skew/min-cost for various number of splits - k (n=100, rand distribution)