Classifying Human Dynamics Without Contact Forces
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Abstract

We develop a classification algorithm for hybrid autoregressive models of human motion for the purpose of video-based analysis and recognition. We assume that some temporal statistics are extracted from the images, and we use them to infer a dynamical system that explicitly models contact forces. We then develop a distance between such models that explicitly factors out exogenous inputs that are not unique to an individual or her gait. We show that such a distance is far more discriminative than the distance between simple linear systems, where most of the energy is devoted to modeling the dynamics of spurious nuisances such as contact forces.

1. Introduction

The analysis of human motion has been a subject of interest in the vision community for decades, further reinforced in recent years by applications in security, biomechanics and entertainment. All aspects of the problem, from modeling to detection, tracking, classification, and recognition are the subject of active research [13, 31]. From a modeling perspective, humans are physical objects interacting in physical space in ways that are mediated by forces, masses and inertias that can be described, to first approximation, by ordinary differential equations. In other words, humans are dynamical systems. Analytically, each individual can be described by a model that includes intrinsic parameters (masses, inertias), internal *states* (skeletal configurations, internal forces), also a property of the individual, and external forces (*inputs*), including contact forces, that depend on the environment and other nuisance factors. From the point of view of perception, humans and their clothes interact with light and an imaging device to yield *output* images.

While "static" (e.g. pose, skeletal configurations [21]), "quasi-static" (e.g. graphs of transitions between poses [29], cumulative video statistics [5]), or "kinematic" representations [6] already contain significant information on both the identity of humans and their action,[1] *dynamics* also play a crucial role, that has been recognized early on by Johansson [16] who showed that even if we strip the image of all of its pictorial content and look at displays of moving dots, from their motion we can often tell whether a person is young or old, happy or sad, man or woman.[2] In this paper we concentrate on *dynamics as a perceptual cue for human motion recognition*.[3]

If we agree in viewing humans as dynamical systems, then learning their dynamic characteristics is a system identification task [23]. System identification is a well established field, and yet in almost 50 years of research the problem of performing decision tasks, such as detection and recognition, in the space of dynamical models is largely unexplored. Several attempts have been made to endow the space of dynamical models with a metric and probabilistic structure, such as the Gap metric [34], subspace angles [10], Martin's distance [25]. However, even for simple linear systems deciding "how far" two models are is not straightforward, and learning a distribution (e.g. a prior) in model space is even less so [19].[4] In particular, if we want to be able to learn models that have discriminative power, we have to factor out nuisance factors, such as external forces, that do not depend on the particular individual or gait. Therefore, in this work we consider *models that explicitly represent*

[1] It is often easy to tell that someone is running, rather than walking, from a single snapshot.

[2] One could argue that moving dot displays also contain pose and kinematic information; however, dynamics remains an important cue, as one can guess by watching two-hundred pound imitators display Charlie Chaplin's walk (different masses, inertias and skeletal configuration, same perceptual dynamics). Furthermore, one single snapshot of such moving dot displays rarely yields much information.

[3] This does not mean that kinematics, or pose or even pictorial cues are not important, and eventually all will have to be integrated into a coherent system. We believe, however, that dynamics has been largely unexploited, hence our emphasis in this paper.

[4] Note that each of these techniques has been applied to the analysis and classification of human motion ([4] for subspace angles and Martin's distance, [26] for the Gap metric) with encouraging but limited results.

contact dynamics; such models are *hybrid*, in the sense that they involve both continuous dynamics and discrete "switches." Therefore, the simplest instance of our problem involves performing *inference and classification of hybrid dynamical models.* Since the analysis is complex enough for *repetitive gaits* (e.g. walking, running, jumping), we concentrate on this case. Ideally an individual should be recognized regardless of the gait, and in particular during transient maneuvers, but this is beyond the scope of this paper.

In order to distill the essence of the problem, we concentrate on dynamics, and assume that some representation of a human gait has been inferred, either in the form of joint angles in a skeletal model (e.g. [7]), or in the form of joint positions, e.g. from a motion-capture system. In other words, we use data similar to Johansson's displays, that distill dynamic information. Note that, although we assume that the "image-to-model" problem is solved, which is not quite the case even today, and although we do not use any images in this work, this is vision work indeed. In fact, the models we study are designed and analyzed for the purpose of vision-based classification: If we were to infer and analyze models for, say, computer graphics, or robotics, or biomechanics, the models would be quite different, and their inference would likely entail additional measurements (e.g. forces) that are not directly available in a vision context. So, we concentrate on *inference and classification of hybrid dynamical models designed for vision-based human motion analysis and recognition.* This is not a trivial problem, and even some of the basic ingredients are missing from the literature, as we explain in the following section.

1.1. Relation to previous work

The literature on human motion recognition is too broad for us to review here. We will provide a synthetic overview of the main approaches, both for the problem of classification of motion gaits [31] and of identification of people by their gait [29].

The proposed approaches can be classified as model-based [6, 4, 20, 18] representing the motion as the parameters of a model fitted to the data, or holistic [22, 33], where some statistics is extracted from the video sequence and used for classification. In all cases the first step consists in deriving a compact representation of the motion, such as binary silhouettes [29, 18], optical flow [22], joint angles of an articulated body model with image-based tracking [6, 4, 27], or other spatio-temporal motion descriptors [3, 12] . Then some statistics are computed on the reduced data and pattern recognition techniques such as PCA [3], bilinear models [20], Hidden Markov Models [**?**, 18], K-Nearest Neighbor [22] or Support Vector Machines [21] are applied to the classification problem.

As we motivated in the introduction, we take the approach of modeling the dynamics of human gaits with hybrid linear models. Inference of the state and model parameters for a switching linear model is, in general, NP complete [32]. While several heuristic algorithms exist (e.g. [35, 1]), there is no optimal algorithm of reasonable complexity for the model orders that we need to consider. Therefore, we concentrate on a specific class of models, that is switching autoregressive (AR) ones. These are a subclass of switching linear system that is particularly attractive since, for each mode, the optimal estimator can be written as a closed-form function of the data [23]. For hybrid-AR models, recent algebraic approaches to filtering and identification [24] have shown promising results, however they do not provide probabilistic information on the estimates and therefore are not suited to our purposes. We will derive our own identification algorithm in Sect. 2.2, and this is our first contribution.

Our second challenge is to define a distance in the space of hybrid-AR models. To the best of our knowledge, this has only been done once before [11] for the case where the models are represented by deterministic unknown parameters, rather than having a distribution of them. We show that the simple extension of [11] to a stochastic model yields non-sensical distances that either are non-zero when the two models are identical (eq. 6), or that can be infinity for models that are arbitrarily close in the deterministic sense (eq. 7). We propose a notion of discrepancy that is very intuitive because it ends up coinciding with the Euclidean distance between the optimal estimators.

The main achievement of this paper is to show that *the distance between hybrid models is far more discriminative than the distance between linear models that was previously used to classify gaits based on their dynamics.* While this may not be surprising at first, since hybrid-AR models are a super-class of linear models, and therefore they naturally have more modeling power, note that discriminative power usually decreases with model complexity, since we can have orbits of model parameters that yield the same output statistics. This is not the case in our model, and we show that it sharply classifies gait data where linear models yield total confusion.

2. Modeling human dynamics for classification

2.1. Autoregresssive Models

Consider a Gaussian linear time-invariant autoregressive (AR) model of order n:

$$y_t = \sum_{i=1}^{n} A_i y_{t-i} + e_t \;\; y_t \in \mathbb{R}^p \;\; e_t \sim \mathcal{N}(0, R) \tag{1}$$

The equation can be rewritten in normal form:

$$y_t = \varphi_t \theta + e_t \tag{2}$$

$$\begin{aligned} \varphi_t &= \left[\begin{array}{cccc} y_{t-1} \otimes I_p & y_{t-2} \otimes I_p & \cdots & y_{t-n} \otimes I_p \end{array} \right] \\ \theta^T &= \left[\begin{array}{cccc} \theta_1^T & \theta_2^T & \cdots & \theta_p^T \end{array} \right] \\ \theta_i^T &= \left[\begin{array}{cccccccc} A_1(i,1) & \cdots & A_1(i,p) & \cdots & A_n(i,1) & \cdots & A_n(i,p) \end{array} \right] \end{aligned}$$

where $\otimes$ denotes the kronecker tensor product and I_p is the identity matrix of dimension p.

Parameter estimation

Assuming Gaussian prior on the parameters: $\theta \sim \mathcal{N}(\theta_0, P_0)$and given a sequence of observations : $y^N = \{y_1, y_2, \cdots, y_N\}$the posterior distribution of the parameters θ is [23]:

$$p(\theta | y^N, \theta_0, P_0, R) = G(\theta; \hat{\theta}, \hat{P}) \tag{3}$$

where:

$$\hat{\theta} = \hat{P}\left(P_0^{-1}\theta_0 + \sum_{t=1}^{N} \varphi_t R^{-1} y_t\right) \;,\; \hat{P} = \left(P_0^{-1} + \sum_{t=1}^{N} \varphi_t R^{-1} \varphi_t^T\right)^{-1} \tag{4}$$

and $G(\theta; \hat{\theta}, \hat{P})$ is the Gaussian density with mean $\hat{\theta}$ and variance $\hat{P}$ evaluated at θ:

$$G(\theta; \hat{\theta}, \hat{P}) = (2\pi)^{-\frac{d}{2}} \det(\hat{P})^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\theta - \hat{\theta})^T \hat{P}^{-1} (\theta - \hat{\theta}\right) \tag{5}$$

For an intuitive understanding of these expressions consider the simple case of scalar measurements $y \in \mathbb{R}$. The equation of $\hat{P}$ reduces to: $\hat{P} = \left(P_0 + \frac{\sum_{t=1}^{N} y_t^2}{R}\right)^{-1} = \left(P_0 + (N-1)\frac{\Sigma_y}{R}\right)^{-1}$, where Σ_y is the sample variance of the measurements. The variance $\hat{P}$ is a measure of the uncertainty we have in the estimated parameters. As we could expect, it decreases as the length N of the observation sequence and the signal-to-noise ratio $\frac{\Sigma_y}{R}$ increase. In the limit $N \to \infty$, the variance $\hat{P}$ becomes zero and the estimate $\hat{\theta}$ is the true value of the parameters.

Model distance (AR)

We use the posterior distributions $p(\theta|y^N)$ on the parameters to define a distance between models. As a first attempt we consider the expectation of the Euclidean distance between the parameters $\theta_1|y_1^N \sim \mathcal{N}(\hat{\theta}_1, \hat{P}_1), \theta_2|y_2^N \sim \mathcal{N}(\hat{\theta}_2, \hat{P}_2)$:

$$\begin{aligned} d_e(\theta_1, \theta_2)^2 &= E[(\theta_1 - \theta_2)^T (\theta_1 - \theta_2)] = \\ &= (\hat{\theta}_1 - \hat{\theta}_2)^T (\hat{\theta}_1 - \hat{\theta}_2) + \text{Trace}(\hat{P}_1 + \hat{P}_2) \end{aligned} \tag{6}$$

Unfortunately, this is not a distance; in particular, it is easy to see that $d(\theta_1, \theta_1) \neq 0$. A second attempt is to consider the symmetric Kullback-Leibler divergence (K-L) between the two distributions:

$$KL(p_1||p_2) = \int p_1(x) \log \frac{p_1(x)}{p_2(x)} + p_2(x) \log \frac{p_2(x)}{p_1(x)} dx \tag{7}$$

which for our Gaussians becomes:

$$KL(\theta_1||\theta_2) = \frac{1}{2}\text{Trace}\left(\hat{P}_2^{-1}\hat{P}_1 + \hat{P}_1^{-1}\hat{P}_2 - 2I\right) + \\ +(\hat{\theta}_1 - \hat{\theta}_2)^T\left(\hat{\Sigma}_1^{-1} + \Sigma_2^{-1}\right)(\theta_1 + \theta_2)$$

The problem with this approach is that as the variances $\hat{\Sigma}_1, \hat{\Sigma}_2$ go to zero (i.e. the confidence on the parameter estimates increases), divergence goes to infinity. This happens because K-L is a measure of the extent to which two probability distributions agree. If the two distributions have no common support the K-L distance is infinite indipendently of how far the distributions are. Such a condition is met for example when we have good estimates from sequences generated by models with different underlying parameters.

We can overcome these problems by using a metric between probabilites distributions known with several names, as the Wasserstein distance, the Mallows distance, the Ornestein distance, or the rho-bar distance. Using the L_2 metric, it is defined between two densities P and Q as :

$$d_W(P,Q)^2 = \inf_F\{E_F[(X-Y)^T(X-Y)] : \\ (X,Y) \sim F, X \sim P, Y \sim Q\} \quad (8)$$

where the infimum is taken over all the joint densities F which have marginals equal to P and Q. This distance represents the solution to the Monge-Kantorovich mass transfer problem, and can be interpreted as the mininimum amount of work that is required to transport a mass of soil with distribution P to an excavation having distribution Q. For Gaussian distributions d_W can be computed analytically as [9]:

$$d_W(\mathcal{N}(\hat{\theta}_1, P_1), \mathcal{N}(\hat{\theta}_2, P_2))^2 = (\hat{\theta}_1 - \hat{\theta}_2)^T(\hat{\theta}_1 - \hat{\theta}_2) + \\ \text{Tr}(P_1 + P_2 - 2(P_1P_2)^{\frac{1}{2}}) \quad (9)$$

This measure has some desirable properties. First it is a proper distance, in particular it satisfies the triangular inequality. This guarantees that if the estimated densities $\hat{P}, \hat{Q}$ are good (i.e $d(P,\hat{P})$ and $d(Q,\hat{Q})$ are small), also the estimated distance $d(\hat{P},\hat{Q})$ is close to the true distance $d(P,Q)$: $|d(P,Q) - d(\hat{P},\hat{Q})| \leq |d(P,\hat{P})| + |d(Q,\hat{Q})|$. Second, it is equal to the Euclidean distance in the case of deterministic distributions $P_1 = P_2 = 0$.

For discrete distributions, the Wasserstein distance is equivalent to the Earth's movers distance, a metric commonly used for measuring texture and color similarities.

In the general case of mixture of Gaussian distributions no close form solution is available an approximation must be used, as we will show in the next section.

2.2. Hybrid Autoregressive Models

In order to properly model contact forces in human motion we follow the approach of [2] in using hybrid models where the switches correspond to ground contacts. However, unlike [2], we intend to use such models for classification, and therefore we introduce a different, and to the best of our knowledge novel, switching autoregressive model. This has some similarity with the Autoregressive HMM proposed in [17], although for each autoregressive model we consider the distribution of the observations y_t for finite length sequences instead of using the asymptotic distribution of $y_t, t \to \infty$.

Consider a discrete Markov chain with m states, transition matrix M and prior probabilities $\pi^m = [\pi_1, \cdots, \pi_m]$. To each state q is associated an AR model with noise covariance R_q and parameter θ_q with prior distribution $\theta_q \sim \mathcal{N}(\theta_{0,q}, P_{0,q})$ The equations of the system are:

$$\begin{aligned} y_t &= \varphi_t\theta_{q_t} + e_{q_t} \quad , \quad e_{q_t} \in \mathcal{N}(0, R_{q_t}) \\ p(q_t|q_{t-1}) &= M(q_t, q_{t-1}) \quad , \quad p(q_1) = \pi_{q_1} \end{aligned} \quad (10)$$

A graphical representation of this model is shown in figure 1. As we can see from the figure, the AR parameters $\theta^m = \{\theta_1, \cdots, \theta_m\}$ are time-invariant random vectors, and the observed outputs y_t induce a distribution on hidden states q_t and model parameters θ^m. The motivation of this model is that we assume m undelying autoregressive models, whose parameters θ_i are random but fixed, and the transitions between models are determined by the hidden states q_t.

In other hybrid AR systems proposed in the literature [11], the parameters θ are modeled as unknown deterministic values. A learning algorithm is derived to compute the maximum likelihood estimate $\theta^{ML} = \arg\max_\theta p(y^N|\theta)$given an observation

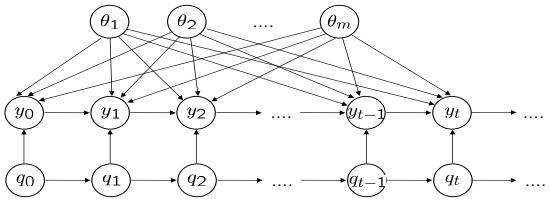

Figure 1. *Dynamical Bayesian network representing our proposed hybrid autoregressive model. Nodes are random vectors and edges are conditional dependence relations. The picture clearly shows the presence of multiple loops in the graph which make exact inference a computationally intractable problem.*

sequence y^N. Unfortunately, this method does not provide a natural way to compare parameters of two models θ_1, θ_2, and a common solution [11] is to use the Euclidean distance between the parameters, $||\theta_1 - \theta_2||$. Our approach is different in the sense that we treat θ as a random vector with given prior distribution $p(\theta)$ and compute the posterior given the observations $p(\theta|y^N)$. This allows us to consider multiple model hypotheses by inferring (multimodal) posteriors on the model parameters and comparing models by using distances between these probabilities distributions.

We can relate the two approaches by considering the case of flat prior $p(\theta) \simeq$ const. Then the posterior $p(\theta|y^N)$ is proportional to the likelihood $p(y^N|\theta)$, and the maximum likelihood estimate is also the maximum a posteriori $\theta^{ML} = \hat{\theta}$. The distance $d^{ML} = ||\hat{\theta}_1 - \hat{\theta}_2||$ measures how far the principal modes of the posterior distributions $p(\theta_1|y^N)$ and $p(\theta_2|y^N)$ are. In the case of hybrid models this solution is suboptimal since the posteriors $p(\theta_i|y^N)$ are typically multimodal mixtures, as we can see in figure 4, while the distance d^{ML} takes into account only one parameter hypothesis.

Parameter Estimation

Given an observation sequence y^N we want to estimate the posterior distribution:

$$p(\theta|y^N,\Lambda) = \sum_{q^N} p(\theta|q^N, y^N, \Lambda) p(q^N|y^N, \Lambda) \; =$$

$$= \sum_{q^N} \sum_{i=1}^{m} p(\theta_i|q^N, y^N, \Lambda) p(q = i|q^N) p(q^N|y^N, \Lambda) \tag{11}$$

where $\Lambda = \{\theta_0^m, P_0^m, R^m, M, \pi\}$ are the model parameters, with $\theta_0^m = \{\theta_{0,1}, \cdots, \theta_{0,m}\}$, $P_0^m = \{P_{0,1}, \cdots, P_{0,m}\}$, $R^m = \{R_1, \cdots, R_m\}$. Similarly to (3), we have that $p(\theta_i|q_j^N, y^N, \Lambda) = G(\theta_i; \hat{\theta}_i, \hat{P}_i)$ are Gaussian, $p(q = i|q^N)$ is the relative frequency of state i in the sequence q^N, and $p(q^N|y^N, \Lambda)$ is the posterior of the hidden states given the observations, can be computed in closed form and will be given in the next section. Unfortunately, marginalizing the hidden states $q^N = \{q_1, \cdots, q_N\}$ is intractable because it would require evaluating an exponential number of hypotheses.

A possible approach to inference would be to apply Gibbs sampling to obtain sequences of hidden states q^N and model parameters θ^m distributed according the posterior. However, we have observed that for this model the parameters typically have highly peaked multimodal distributions, which make likely for the Gibbs sampler to be trapped in local modes and thus would require to draw a large number of samples to obtain good approximations.

On the other hand, the graphical model in figure 1 shows that each parameter θ_i is statistically dependent on all the observations y_t. These would make convergence problematic for inference algorithms such as loopy belief propagation.

We could think of applying variational inference techniques in order to obtain an approximate model with a smaller number of dependencies for which the inference problem would be easier. Typically these methods work by approximating the posterior of the hidden variables q^N given observations y^N and parameters θ^m. However notice that by doing so there is no simple way to break the dependencies between outputs and parameters, therefore we would not remove the main source of complexity in the model.

Then our solution is to approximate the posterior using a bank of K filters, where each filter is tuned on a segmentation hypothesis q_j^N. At each time t we generate a new hypothesis q_t^N by imposing a jump to the most likely sequence and discarding the less likely ones. We approximate the posterior with:

$$p(\theta|y^N, \Lambda) \simeq \frac{1}{C} \sum_{j=1}^{K} \sum_{i=1}^{m} p(\theta_i|q_j^N, y^N, \Lambda) p(q = i|q_j^N) p(q_j^N|y^N, \Lambda) \tag{12}$$

where $C = \sum_{j=1}^{K} p(q_j^N|y^N, \Lambda)$ and q_j^N are the filter hypotheses. Therefore this approximation is a mixture of a constant number Km of Gaussians. In practice we have duplicate hypotheses (due to permutation of the states) and hypotheses with low posterior, so the effective number of components is smaller (figure 4).

Hidden State Filtering

In order to obtain a good approximation of the posterior (12) we need to estimate the K most probable hidden state sequences $q_1^N, \cdots, q_K^N$ given the measurements y^N:

$$\hat{q}_1^N, \cdots, \hat{q}_K^N = \arg \max_{q_1^N, \cdots, q_K^N} \sum_{i=1}^{K} p(q_i^N | y^N, \Lambda) \tag{13}$$

We can derive a recursive equation for the posterior up to time t:

$$p(q^t | y^t, \Lambda) = \frac{K_{t-1}}{K_t} p(y_t | q^t, y^{t-1}, \Lambda) p(q_t | q_{t-1}, \Lambda) p(q^{t-1} | y^{t-1}, \Lambda)$$

where $K_t = p(y^t|\Lambda)$ is a constant independent of q^t, $p(q_t|q_{t-1}, \Lambda) = M(q_{t-1}, q_t)$, $t > 1$ and $p(q_1|q_0, \Lambda) = \pi_{q_1}$. Substituting $p(y_t|q^t, y^{t-1}, \Lambda) \sim \mathcal{N}(\varphi_t^T \theta_{q_t,t-1}, \varphi_t^T P_{q_t,t-1} \varphi_t + R_{q_t})$, and taking the logarithms, we have:

$$\begin{aligned} \log p(q^t | y^t, \Lambda) = \log p(q^{t-1} | y^{t-1}, \Lambda) + C + \\ + \log M(q_{t-1}, q_t) - \frac{1}{2} \log \det \Gamma - \\ - \frac{1}{2} \left(y_t - \varphi_t^T \hat{\theta}_{q_t,t-1} \right)^T \Gamma^{-1} \left(y_t - \varphi_t^T \hat{\theta}_{q_t,t-1} \right) \end{aligned} \tag{14}$$

where C is a constant, $\Gamma = \left(\varphi_t^T \hat{P}_{q_t,t-1} \varphi_t + R_{q_t} \right)$ and $\hat{\theta}_{i,t}, \hat{P}_{i,t}$ are the estimates at time t of the parameters θ_i associated to state i (compare to (4)):

$$\hat{\theta}_{i,t} = \hat{P}_{i,t} \left(P_{0,i}^{-1} \theta_{0,i} + \sum_{j | q_j = i, j \le t} \varphi_j R_i^{-1} y_j \right) \tag{15}$$

$$, \ \hat{P}_{i,t} = \left(P_{0_i}^{-1} + \sum_{j | q_j = i, j \le t} \varphi_j R_i^{-1} \varphi_j^T \right)^{-1} \tag{16}$$

To find the most probable state sequences (13) we use a bank of K filters, each matched to a hidden state sequence hypothesis q_i^N, and the posterior $p(q^N|y^N, \Lambda)$ is computed recursively with (14). We initialize the filters at $t = 1$ so that there is at least one hypothesis $q_{i,1} = i$ for each possible initial state value $i = \{1, \cdots, m\}$. Then for each time $t = 2, \cdots, N$ we iterate the following rules to maintain the K hypothesis:

- For each hypothesis q_i^t, compute the posterior loglikelihood $\log p(q_i^t|y^t)$ using (14).
- Extend the hypotheses $q_j^t, j = 1, \cdots, K$ to $t+1$ by assuming no switch: $q_j^{t+1} = \{q_j^t, q_{j,t}\}$
- Let the most probable sequence q_o^t split at time $t+1$, i.e. generate $m-1$ new hypotheses q_{K+i}^{t+1} such that $\{q_{K+i,t+1}\} = \{1, \cdots, m\} \backslash \{q_{o,t}\}$.
- Cut off the $m-1$ least probable sequences, so only K are left.

This algorithm exploits the finite memory property of our hybrid model. Past data do not contain information on what happens after a switch, therefore only the most likely sequence among all with a switch at a given time has to be considered. The number of filters K determines the quality of the estimates. With $K \ge N$, the algorithm is guaranteed to find the optimal state sequences (13) [14]. In order to improve the performances it is useful to assume a minimum segment length l and allow splitting and cut off only for sequences that did not switch in the last l steps.

Distance between Hybrid AR models

We obtain a discrepancy measure between models by extending to hybrid models the distance (9) between posteriors of autoregressive parameters. Let the posterior distribution of the parameter θ_k be

$$p(\theta_k | y_k^N, \Lambda) = \sum_{i=1}^{n_k} \alpha_{k,i} G(\theta_k; \hat{\theta}_{k,i}, \hat{P}_{k,i})$$

The Wasserstein distance between general mixtures of gaussians cannot be computed in closed form. Following [15], we approximate $d_W(\theta_1, \theta_2)$ by solving a maximum flow problem. We have:

$$d_W(\theta_1, \theta_2) = \frac{\sum_{i=1}^{n_1} \sum_{j=1}^{n} f_{i,j} d_W(\mathcal{N}(\hat{\theta}_{1,i}, \hat{P}_{1,i}), \mathcal{N}(\hat{\theta}_{1,i}, \hat{P}_{1,i}))}{\sum_{i=1}^{n_1} \sum_{j=1}^{n} f_{i,j}} \tag{17}$$

Subject	1	2	3	4	5	6	7	8	9	10	11	Total
Walk	14	14	7	14	14	2	5	14	7	14	9	114
Run	14	8	9	14	8	-	7	3	-	8	8	79
Limp	4	2	4	5	5	-	5	5	-	5	5	40
Total	32	24	20	33	27	2	17	22	7	27	22	233

Figure 2. *List of motion capture data sequences in the gait dataset. For each subject (first column), number of walking, running and limping sequences collected.*

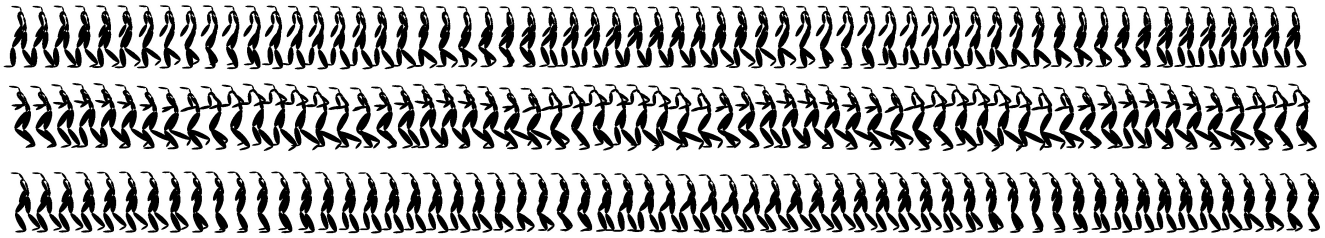

Figure 3. *Short clips (about 3 seconds) from the sequences of the gait dataset. Subject 1 walking (top), running (center) and limping (bottom)*

where the Wasserstein distance between normal distributions is given in (9) and $f_{i,j} \geq 0$ is the optimal admissible flow that satisfies the constraints:

$$\sum_{j=1}^{n} f_{i,j} \leq \alpha_{1,i} \quad , \quad \sum_{i=1}^{n_1} f_{i,j} \leq \alpha_{2,j} \quad , \quad \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} f_{i,j} = 1$$

In the next section we will use this distance to compare hybrid dynamical models learned from human motion data.

3. Experiments

Our goal in this research is to recognize human motion based on dynamic signatures. We believe that dynamics contain a significant amount of information: Johansson's stripped-down moving-dot displays [16] can allow one to infer whether the person is young, old, happy, sad, even man or woman, which is information likely not coded in the pose or configuration. In particular, we use a hybrid dynamical model because we have determined that the contact dynamics, which is an exogenous event independent of the individual and her gait, is a dominant dynamic event that must be factored out of the classification and recognition process. However, our framework applies to the recognition of dynamic events in general, without restriction to human motion. In particular, even within human motion, our framework applies to different representations, from the trajectories of moving intensity blobs, to the joint angles estimated from a video-based tracking system, to the position of retro-refractive markers in motion capture.

In the specific case described in this section, the data used in the experiments is given as a set of joint angle trajectories on a skeletal model of the human body. These angles may be obtained from a video-based full body tracker or from a motion capture system. We opted for the latter for ease of collection and ground-truth testing. We used a 6-camera infrared motion capture system running at 60Hz; we used 20 retro-reflective markers on the test subjects at the proximity of the body joint locations and recorded the marker trajectories during the motion. The subjects were asked to walk, run and limp on a treadmill. We collected a total of 233 sequences from 11 subjects, see table 2 for details. Each sequence is sampled at 60 Hz and is about 6 second-long.

From marker positions we estimated body skeleton model and join angles with an approach similar to the one proposed in [28]. First we estimate the reference frame moving with the body limb from the set of markers attached to the limb. Then the joint positions are obtained as the center of rotation of the reference frames of adjacent limb. From joint positions, by enforcing fixed limb length we obtained skeleton model and joint angles. Since we do not use a reference model for the skeleton, the estimated skeletons vary from person to person, this affecting the joint angles estimates and making the recognition problem harder. In figure 3 we show some sample clips of the data sequences. Of course we could use pose and configuration information to aid the classification, since that is available in our dataset. However, we are not going to do so because (a) these data are generally not reliable when estimated directly from video, and (b) although pose and configuration are important, many groups are addressing them, and we therefore want to focus our attention on dynamics. Naturally, eventually all will have to be integrated into a complete classification system.

From each sequence, we extracted the 24 angles corresponding to the 8 joints defining the positions of hips, femurs, tibias

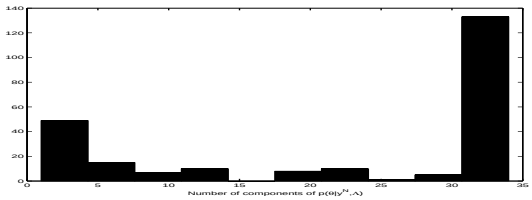

Figure 4. *Histogram of the number of components of the parameter posterior* (12) *for the gait sequences in the dataset. These results show that we cannot assume the posterior to be unimodal.*

and feet. The angles are expressed in the exponential map parameterization. Since the number of parameters of the AR model is p^2, where p is the dimension of the measurements, we had to reduce the dimensionality of the data. For this purpose we applied PCA to each sequence, and retained the first $p = 4$ components. Given a sequence y_i^N we learned the posterior (12) using the algorithm described in section 2.1. The model we propose is very general and contains a number of parameter that should to be tuned to the particular class of signals under investigation. In these experiments, we used first-order autoregressive models, i.e. $n = 1$ in (1). We set prior means θ_0^m to zero and the prior variances P_0^m to $p_0 I$, where p^0 is a large number, thus modeling the lack of prior information on the parameters. The noise variances R^m are set to the identity, so that in (15) we obtain least squares estimates. We have 2 hidden states and the Markov chain parameters M, π are so that all state have equal probability and the average length of a segment is L: $M(i,i) = \frac{L}{L+1}$, $M(i,j) = \frac{1}{(L+1)(m-1)}$ $i \neq j$, $\pi_i = \frac{1}{m}$. The posteriors are computed with a bank of K filters. To have optimal segmentations we would need K to be not smaller than the sequence length N, typically about 400. In practice we noticed reducing K to 50 does not change significantly the approximation (12). Since some of the computed segmentation hypotheses are equivalent (they are equal up to a permutation of the states), the filtering is followed by a hypothesis reduction step where we remove the duplicate hypotheses. Then we proceed to computing the posterior on the parameters (12). Of all the components of (12), typically only few of them have weight significantly different form zero. Therefore we proceed to pruning all the hypothesis that have weight below a small threshold. In figure (4) we show the histogram of the number of components of the posteriors learned from the gait sequences. We see that most of the sequences have multimodal distribution, with number of modes limited by the number of filters K.

3.1. Hybrid models for dynamic discrimination

The point of this section is to show that hybrid models have more discriminative power than simpler linear models [4]. Our intent here is to show that discrimination between different classes (e.g. different gaits by the same individual, or different individuals walking the same gait) is made possible by a hybrid model where it was not by using a linear dynamical model.

This is, therefore, a feasibility study, and we do not need to compete with other gait or individual recognition techniques that use different (static) features. Our approach is meant to complement them, not to replace them.

In figure (5) we show the pairwise distance between models learned from the dataset sequences. We clearly see that the hybrid models can discriminate between gait classes. For comparison, we learned first order autoregressive models from the same sequences and computed the Euclidean distance between the maximum likelihood parameter estimates. By using this simpler model measure we would not be able to discriminate between gaits. The confusion between limp and walk may be due to the different parameterizations of the motion, to the dimensionality reduction step or simply to the fact that the dynamics of the two gaits are very close.

4. Discussion

We have presented a technique to perform classification in the space of hybrid autoregressive models that we have used to classify human gait. We have shown that classification based on a hybrid model yields significant improvements over simple linear systems.

In order to achieve our results, we had to devise a novel (approximate) filtering and identification technique for hybrid AR models (this is inspired by a wealth or results available in the literature), and introduce a distance between parameter distributions. This distance is not computable efficiently, so we had to resort to an approximation, which nonetheless showed good performances in our experiments.

Our results are restricted to *stationary (quasi-periodic) gaits*. Ideally we would like to recognize transient actions, but doing so in a principled manner is beyond our means at the moment, so we prefer to concentrate on a simpler problem. We also assume, somewhat optimistically, that temporal statistics are extracted for us from images. This does not mean that we under-appreciate the difficulty in detecting, localizing, and tracking humans in video, on the contrary. The models we propose

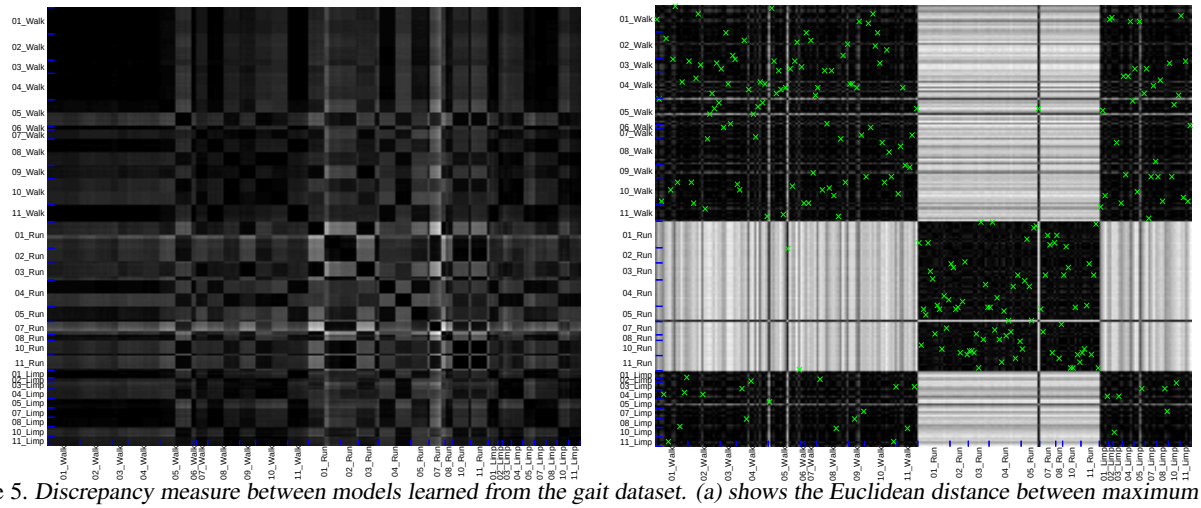

Figure 5. *Discrepancy measure between models learned from the gait dataset. (a) shows the Euclidean distance between maximum likelihood estimates of autoregressive model parameters. (b) displays the approximated Wasserstein distance between posterior distributions of the parameters of hybrid autoregressive models. We can see that the simple autoregressive models are not discriminative enough to capture the character of the motion class. It appears that the limping and walking gait are not successfully discriminated. This is not surprising: since it is hard to limp on a running treadmill, the dynamics of the two gaits, as we see in figure 3, are very close.*

can be used to *support* these tasks, eventually, and our inference techniques relies on a model that is inferrable from images. This is not, therefore, a paper in graphics, since it seeks models with discriminative power, not with generative power. We do not assume that forces or higher-order temporal statistics are available, which would be the case if we were analyzing data for graphics or biomechanics.

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