A Graphical Solution to Statistical Adjustment

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Abstract

Whenever we undertake to evaluate the effect of one factor (X) on another (Y), the question arises as to whether we should adjust our measurements for possible variations in some other variable, Z, sometimes called "covariate", "concomitant" or "confounder". Adjustment amounts to partitioning the population into groups that are homogeneous relative to Z, assessing the effect of X on Y in each homogeneous group and, finally, averaging the results. Despite a century of statistical analysis, the basic question – whether an adjustment for a given covariate Z is appropriate in any given study – continues to be debated informally, on a cases by case basis, resting on folklore and intuition rather than on hard mathematics. This paper reports a formal and general solution to the problem of statistical adjustment using the language of graphs.

Whenever we undertake to evaluate the effect of one factor (X) on another (Y), the question arises as to whether we should adjust our measurements for possible variations in some other variable, Z, sometimes called "covariate", "concomitant" or "confounder". Adjustment amounts to partitioning the population into groups that are homogeneous relative to Z, assessing the effect of X on Y in each homogeneous group and, finally, averaging the results. The importance of such adjustment has been recognized as early as 1898, when Karl Pearson, the founder of modern statistics, discovered what in modern terms would be called the Simpson's paradox: Any statistical relationship between two variables may be reversed or negated by including additional factors in the analysis.

The classical case demonstrating Simpson's reversal is the study of Berkeley's alleged sex bias in graduate admission [Bickel *et al.*, 1975], where overall, data show a higher rate of admission among male applicants but, broken down by departments, data show a slight bias toward female applicants.

Despite a century of analysis, the Simpson's reversal phenomenon continues to "trap the unwary" [Dawid, 1979] and the main question – whether an adjustment for a given covariate Z is appropriate in any given study – continues to be debated informally, on a cases by case basis, resting on folklore and intuition rather than on hard mathematics. The statistical literature is remarkably silent on this issue and, aside from the standard advice that

one should not adjust for a covariate that is affected by the putative cause (X), it provides no further guidelines as to which covariates would be admissible for adjustment and what assumptions would be needed for making this determination. In the Berkeley story, for example, if we suspect that a certain undisclosed attribute of a candidate (e.g., music aptitude) has influenced both the admission decision and applicants' choice of departments, then it is not quite clear whether adjusting for department choice would reduce or increase bias in assessing alleged sex discrimination in admissions. More significantly, it is not uncommon to find trained statisticians disagreeing on such issues, even if the confounding attribute can safely be assumed gender-independent. A concrete controversy that arose out of the problem of covariate adjustment is the episode of "reverse regression" which occupied the social science literature in the 1970's: Should we, in salary discrimination cases, compare salaries of equally qualified men and women or, instead, compare qualifications of equally paid men and women. Another controversy, known as the Lord Paradox, originated with the question of whether we should adjust for socioeconomic status in assessing the effect of tutoring program on reading score, knowing that children from well to do families, who do better on the reading test, are more likely to be in the program. Lord paradox, in its various manifestations, still lingers on, in the psychometric literature [Wainer, 1991].

The primary reason for the persistence of confusion over the appropriateness of statistical adjustment is that the answers depend on causal assumptions, and statisticians, by and large, are reluctant to discuss such assumptions forthrightly. First, causal assumptions (e.g., that the rooster's call does not cause the sun to rise) are perceived less objective than typical statistical assumptions (e.g., that two variables are correlated), presumably because the former cannot be established by observational studies. Second, even when causal facts are established to the satisfaction of statistical standards (say, by subjecting our rooster to controlled randomized experiment), we still lack an adequate mathematical notation to state those assumptions symbolically – causal relationships cannot be distinguished from statistical associations in the standard language of probability theory.

The purpose of this paper is to report a formal and general solution to the problem of statistical adjustment using the language of graphs. This language, which can be traced back to the geneticist Sewal Wright (1919) , permits the investigator to express causal assumptions in the form of arrows among quantities of interest, and, once the graphs are completed, a simple procedure would decide whether a proposed adjustment is appropriate relative to the quantity under evaluation.

The procedure is described in the following five steps, which determine whether a set of variables Z should be adjusted for, when we we wish to evaluate the total effect of X on Y.

Step 1. (simplification) Simplify the diagram by eliminating all nodes (and their incident edges) which are not ancestors of either X, Y or Z.

Step 2. (triangulation) Add an undirected edge between any two ancestors of Z which share a common child.

Step 3. (pruning) Eliminate all arrows emanating from X.

Step 4. (symmetrization) Strip the arrows from all directed edges.

Step 5. (test) If, in the resulting undirected graph, Z intercepts all paths between X and

Y, then Z is an appropriate covariate for statistical adjustment. Else, Z should not be adjusted for.

References

[Bickel *et al.*, 1975] P.J. Bickel, E.A. Hammel, and J.W. O'Connell. Sex bias in graduate admissions: Data from berkeley. *Science*, 187:398–404, 1975.

[Dawid, 1979] A.P. Dawid. Conditional independence in statistical theory. *Journal of the Royal Statistical Society, Series A*, 41:1–31, 1979.

[Wainer, 1991] H. Wainer. Adjusting for differential base-rates: Lord's paradox again. *Psychological Bulletin*, 109:147–151, 1991.

[Wright, 1921] S. Wright. Correlated and causation. *Journal of Agricultural Research*, 20:557–585, 1921.