Computer Science Department Technical Report
University of California
Los Angeles, CA 90024-1596

ON PERFORMANCE BOUNDS FOR TWO RECTILINEAR STEINER TREE HEURISTICS IN ARBITRARY DIMENSION

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May 1990
CSD-900015

On Performance Bounds for Two Rectilinear Steiner Tree Heuristics in Arbitrary Dimension

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Abstract

It was widely believed that the ratio of the length of an optimal minimum spanning tree (MST)-derived rectilinear Steiner tree [2] to the length of the minimum rectilinear Steiner tree (MRST) is bounded away from 3/2. It was also conjectured [1] that a Kruskal-type RST heuristic described in [5] yields solutions with length no more than 5/4 times optimal. We disprove these conjectures by exhibiting a single counter-example for which the performance ratio of each heuristic is arbitrarily close to 3/2. These results are sharp since 3/2 is an upper bound on both performance ratios [4]. The counter-example generalizes to arbitrary dimension D, where the heuristics will have error bound at least (2D - 1)/D.

Introduction

The rectilinear Steiner problem has been extensively studied in VLSI layout because solutions correspond to optimal circuit wiring in the Manhattan L_1 norm. The problem is NP-complete [3], and a number of heuristics have been proposed which are derived from minimum spanning tree (MST) construction methods, or line-sweep methods from computational geometry. Hwang [4] showed that the rectilinear MST is itself an approximation to the MRST with worst-case ratio length(MST) / length(MRST) $\leq$ 3/2. All MRST heuristics proposed thus far have very similar performance on random instances (i.e., average heuristic Steiner tree length being 8-9% smaller than MST length), and have tight worst-case bounds of 3/2, the same as for the rectilinear MST. [5] and [6] survey these results.

Recently, Ho, Vijayan and Wong [2] gave a method for constructing the optimal RST that lies within the union of the bounding boxes of rectilinear MST edges. The average-case performance was similar to those of previous methods, but it was hoped that a worst-case performance bound less than 3/2 could be proved. Another method [5] was conjectured to have worst-case ratio of 5/4 [1]. This note gives an infinite class of inputs for which these two methods have performance ratio 3/2. Thus, it is still open whether any MRST heuristic exists with better worst-case ratio than the simple MST approximation.

The Counter-Example for 2 Dimensions

An appealing heuristic for rectilinear Steiner tree construction involves starting with a fixed rectilinear MST, and from it computing the best rectilinear Steiner tree that is restricted to lie within the union of the bounding boxes of its edges. Such a tree will obviously be at least as good as the MST. The authors of [2] gave the surprising result that this optimal MST-derived RST can be found in linear time. Several workers conjectured that the worst-case error of this method was less than 3/2, and in fact, equal to 4/3. We give a counter-example (Figure 1) where this ratio is exactly 3/2, and this result is sharp by Hwang's theorem.

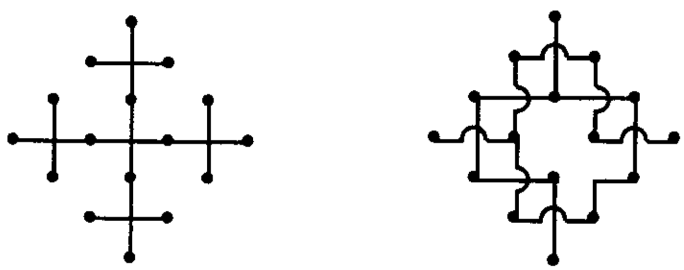

Figure 1: An example where the ratio of an MST-derived-RST/optimal-MRST = 3/2. On the left is the optimal MRST (cost 20); any Steiner tree derived from the MST on the right will have a cost of 30.

Even if we insist that the starting MST be "separable" in the sense of [2], i.e., that its edges are not allowed to intersect except at their end-points, the ratio 3/2 can still be approached arbitrarily closely, as shown in figure 2.

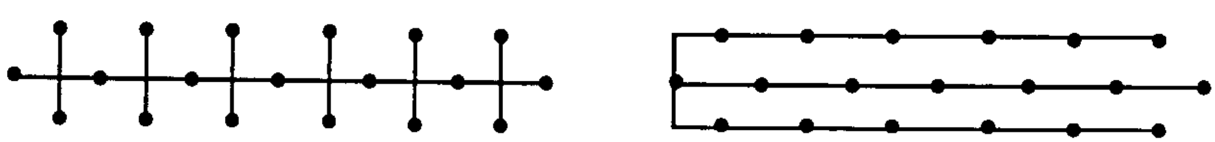

Figure 2: An example where the ratio of a SEPARABLE-MST-derived-RST to the MRST is arbitrarily close to 3/2. For the pointset shown, the MRST on the left has a cost of (4/3)(N-1), while any RST derivable from the MST on the right has a cost of 2(N-2), yielding a ratio arbitrarily close to 3/2 for large enough N.

Interestingly, the example of Figure 2 also shows that a "folklore" heuristic described in [5] and [1] (and the Ph.D. work of the author of [1]) also has worst-case performance ratio arbitrarily close to 3/2. This heuristic consists of growing Steiner-connected components and joining the pair of closest components at each step in such a way as to keep the orientation of each L-corner ambiguous until some connection forces it to become fixed. This is basically a variant of Kruskal's MST method which localizes edges as late as possible. When this heuristic runs on the pointset of figure 2, it may start at one end and alternate between the top, middle and bottom levels, adding a horizontal segment to each in turn; the resulting Steiner tree will consist entirely of straight line segments, except at the starting end, and its length will be arbitratily close to 3/2 times optimal.

Examples for Arbitrary Dimension

Both heuristics extend easily to higher dimensions and are thus of interest for, e.g., 3-dimensional VLSI routing. However, the example of Figure 2 also generalizes to higher dimensions. For N=(2D - 1)k + 1 for any non-negative integer k, the cost of the optimal Steiner tree is at most 2D(N - 1)/(2D - 1), the cost of a separable MST is 2(N - 1), while the cost of a MRST derivable from a certain seperable MST (illustrated in Figure 3 for D=3) is 2(N-D). Therefore, in D dimensions the ratio of the length of either the optimal MST-derived rectilinear Steiner tree or the Kruskal-type tree to the optimal rectilinear Steiner tree can be arbitrarily close to (2D - 1)/D.

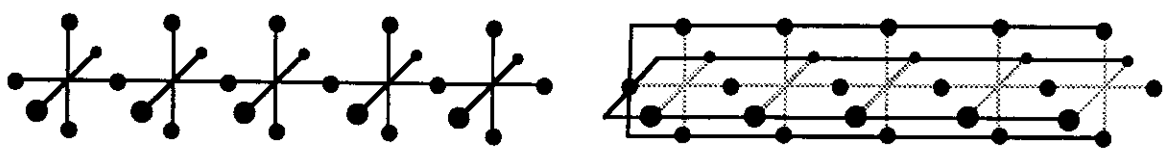

Figure 3: An example for D = 3 where the ratio is arbitrarily close to 5/3. The 3-dimensional MRST on the left has a cost of (6/5)(N-1), while any MRST derivable from the MST on the right, has a cost of 2(N-3), yielding a ratio arbitrarily close to 5/3 for large enough N.

All of the examples given can be slightly perturbed so that the heuristic RST will be unique and still yield the same performance bound. Thus the performance ratios of the heuristics shown here are not artifacts of any ambiguity in the heuristic RST construction.

Open Problems

We conjecture that (2D - 1)/D is a general upper bound for the worst-case ratio (MST-derived-MRST / MRST) in D dimensions, and that (2D - 1)/D is also the higher-dimensional analogue of Hwang's value of 3/2 for D=2. Of course, the basic question remains whether there is an MRST heuristic with worst-case performance ratio less than 3/2.

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