

**COMPREHENSION DRIVEN GENERATION OF
META-TECHNICAL UTTERANCES IN MATH TUTORING**

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**June 1986
CSD-860096**

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ABSTRACT

A technical discussion often contains conversational expressions like “however,” “as I have stated before,” “next,” etc. These expressions, denoted *Meta-technical Utterances* (MTUs) carry important information which the listener uses to speed up the comprehension process. In this research we model the meaning of MTUs in terms of their anticipated effect on the listener comprehension, and use these predictions to select MTUs and weave them into a computer generated discourse. This paradigm was implemented in a system called FIGMENT, which generates commentaries on the solution of algebraic equations.

I INTRODUCTION

When generating tutorial text, a teacher wishes to present the information in the most accessible manner. Clearly, a necessary precondition is that the teacher transmit the appropriate information items. However, we also notice the presence of expressions like “however,” “as I have stated before,” “next,” “generally speaking,” etc., which are not part of the subject matter. These expressions, denoted *Meta-*

*This work was supported in part by the National Science Foundation Grants IST 81-19045 and DCR 83-13875

Technical Utterances (MTUs), carry important information which assists the listener in the assimilation of the transferred knowledge.

Previous research on the semantics of a subset of these utterances (Farnes 1973, Winter 1968, Reichman 1984 and Hoey 1979) indicates that the presence of an MTU can **signpost** what kind of information is to be presented in the forthcoming sentences. Farnes further claims that “the identification and use by readers of such cues, greatly aids comprehension,” and Hoey points out that problems of comprehension have been shown to arise due to faulty or missing signaling.

The text generated by natural language generation systems designed by Davey (1979), Mann and Moore (1980), McKeown (1982), Swartout (1982), Kukich (1984) contains mostly MTUs like “however,” “next” and “therefore,” which directly reflect the speaker’s organization of the subject matter, i.e., they represent the relationship between two or more items of knowledge. For example, if item B violates the expectations established by item A, this relationship is expressed by the utterance: “A, **however** B.” This type of MTUs shall be denoted *Knowledge-Organization MTUs*.

These, however, account only for a fraction of the MTUs found in natural discourse. Teachers often use MTUs such as “as I have stated before,” “let us try another approach,” “in other words” and “this equation is somewhat complicated.” These utterances are more intimately connected with the listener’s learning process than with the organization of the subject matter. This paper describes a generative model of the meaning of these MTUs, based on simulating important aspects of the comprehension process.

In the following section we shall present a functional classification of both types of MTUs. Then we shall describe the system that generates them.

II FUNCTIONAL TAXONOMY OF MTUS

The classification of Meta-Technical Utterances presented in this section is based on their function, as seen by the tutor, in transmitting the subject matter to the student. In our taxonomy we recognize three main functions of MTUs: (1) Knowledge Organization, (2) Knowledge Acquisition and (3) Affect Maintenance.

A. Knowledge-Organization MTUs

The information residing in a tutor's mind can be visualized as a network whose nodes contain individual information items, and whose links contain the relations between the nodes. For example: NODE1 contains the *purpose* of NODE2, or NODE3 is an *alternative* to NODE1. These relations directly reflect the tutor's knowledge about equations and their solution, and they roughly correspond to Hallyday and Hasan's external/internal category (Hallyday & Hasan 1976) and Longacre's basic heading (Longacre 1976). The Knowledge-Organization function consists of transmitting these relations. The following headings rely heavily on the taxonomy performed by Hallyday and Hasan, however they have been adjusted to support text-generation.

Additive - These MTUs signal *additional events* ("and," "also"); *realization of expectations* ("indeed") or *availability of additional alternatives* ("alternatively," "or").

Adversative - These MTUs signal *violation of expectations* ("however," "nevertheless," "although," "but," "despite this"); *dismissal*, which indicates that two

different paths in a solution arrive at the same pattern (“either way”) or *recognition*, which signals that an implicit pattern is recognized and made explicit (“notice that”).

Causal - These MTUs pertain to the knowledge about the subject matter, and signal the *reason for performing an operation* (“therefore,” “so,” “then,” “because of this”); its *purpose* (“for this purpose,” “to this end”); *expectations* (“hopefully,” “expecting to get”); *result* (“as a result,” “in consequence”); *means* (“this can be accomplished by”), or *correctness* (“this works because”).

Attributive - These MTUs signal a *generality* or *particularity* relationship (“in general,” “certain types of”).

Temporal - These MTUs signal *sequence* (“then,” “next,” “finally”) or *partial sequence* (“at the same time”).

B. Knowledge-Acquisition MTUs

The MTUs that perform the Knowledge-Acquisition function are related to the state of the discourse and the interaction between the teacher and the student, rather than to the subject matter itself. They ease the assimilation of the subject matter by alerting the student to prepare adequate mental resources. In the context of tutoring algebra, the Knowledge-Acquisition function is performed by the following types of MTUs:

Motivational - A teacher will often use this type of MTUs to motivate a student to listen to the forthcoming technical utterance. For example, if a new method is to be taught, the tutor might say: “This method is very quick.” If a student has to practice the same type of equation many times, the teacher might say: “Third degree equations

are rather difficult and demand lots of practice.”

Focal - A student generally attempts to process a forthcoming technical utterance in the currently active focus space (Grosz 1977). If the teacher wants the student to change the active focus space, he needs to present the student with an MTU to this effect. For example, the Focal MTU “Let us now consider the following equation” closes the focus space corresponding to the previous equation, and opens a new focus space for the next equation. Temporary focus shifts are signaled by MTUs like “incidentally” or “by the way” (Reichman 1984 and Grosz & Sidner 1985).

Categorical - These MTUs specify the manner in which a student should use the forthcoming information to update information in the previous technical utterance. For example, a tutor might say: “Let’s take the first term on the right hand side, namely $x^2(x-3)$, ...” In this example, the MTU “namely” informs the listener that the explicit term merely paraphrases the preceding positional description and is not to be added to the first term on the right hand side. Categorical MTUs are included in Hallyday and Hasan’s internal category (Hallyday & Hasan 1976) and in Longacre’s elaborative heading (Longacre 1976). Other MTUs in this subclass are: “in other words,” “to be more specific” and “for example.”

Implementational - These MTUs prepare the student to select a computational activity required for assimilating the technical utterance that follows. We have identified two main types of activities: *adding* an item to one’s knowledge pool, and *verifying* the workings of existing knowledge (for possible revision). For instance, if the tutor wishes a student to use the forthcoming technical utterance to verify existing knowledge, he should signal his intent by means of an MTU like “as I have stated before.” On the other hand, if the teacher wants the student to prepare for learning a

new subject (i.e., transfer to addition mode) he might say: “Let us now discuss a new topic.”

Estimational - These MTUs inform the student that the forthcoming technical utterance is of unusual length and/or complexity. Examples are: “This equation is rather straightforward,” or “The following method entails several computations.”

In order to illustrate the importance of Knowledge-Acquisition MTUs, let us examine the following imperfect discourse:

“Let us consider a linear equation”

$$3x-7=4$$

... description of solution ...

“Let us consider a linear equation”

$$2x+4=5$$

... description of solution ...

The dissonance in this discourse stems from the repetition of preparatory directives in lines 1 and 4. Both directives trigger expectations for receiving a new object, while, in fact, the second object is of the same class as the first. The appropriate directive for the fourth line should have been *“Let us consider another linear equation.”*

In order to generate a commentary which includes Knowledge-Acquisition MTUs, we need a module that represents how both technical and meta-technical utterances influence the listener’s mental activities. This module would inspect the technical utterances about to be issued, determine their effect on the comprehension processes of the listener, and generate adequate Knowledge-Acquisition MTUs. We call this module Comprehension-Processes Module (see section III).

C. Affect-Maintenance MTUs

One of the tutor's goals is to teach which algebraic operations and results are considered favorable and which are not. In addition, the tutor wishes the attitude of the student to remain positive throughout the session. To achieve these goals, a tutor may need to use Affect-Maintenance MTUs, which we divide into two subclasses according to their goals.

Affect-Transference - If the tutor is of the opinion that the forthcoming technical utterance should have an affective impact on the student, he might precede it by an MTU like "Unfortunately" or "Fortunately." For example, in the sentence "Unfortunately, the only way of solving this equation is to remove parentheses and collect terms," the affect-transference MTU "unfortunately" indicates that this approach is considered undesirable.

Consolatory - A teacher can partially attain the goal of maintaining a positive student attitude throughout a tutorial session by using Knowledge-Acquisition MTUs. There exist, however, situations in which negative affects cannot be prevented by means of these MTUs. For instance, a student may fail to understand a solution method, despite having received preparatory Knowledge-Acquisition MTUs. In cases like this, a teacher should reassure and console the student. This is the purpose of consolatory MTUs such as "Don't worry, I will explain this a few more times." Unlike affect-transference MTUs, consolatory MTUs are related to the state of the listener's learning process.

To generate commentaries which have a desired affective influence on a student, a discourse generator needs to neutralize anticipated negative affects by generating adequate Affect-Maintenance MTUs.

III DESIGN OF FIGMENT

The system outlined in this section was designed to generate fluent and cogent commentaries on algebraic equations, based on the taxonomy presented in the preceding section. The generation of each commentary is performed in three stages.

In the first stage, the strategic components of FIGMENT produce a *technical file*, which consists of a list of *technical messages* (see figure 1). These components are: (1) Problem-Solving Expert, (2) Model of the Student's Knowledge and (3) Tutoring Strategist (see Sleeman and Brown 1982). The Problem-Solving Expert solves the equation and produces a graph in which each branch contains an attempted solution alternative. Next, the Tutoring Strategist modifies this graph by suppressing alternatives and steps which are well known to the student and adding explanations where necessary (e.g., purpose of an operation, its description, etc). Both modules use information about the state of the student's knowledge provided by a Model of the Student's Knowledge.

In the next stage, the Comprehension-Processes Module complements and revises the technical file by adding appropriate MTUs. The affect-transference MTUs and most Knowledge-Organization MTUs can be directly derived from the structure of the technical file. While the Knowledge-Acquisition and consolatory MTUs are generated by simulating some of the comprehension processes activated by a student when reading or listening to an explanation.

In the final stage, the Sentence Composer organizes the completed message into paragraphs and sentences, and translates it into English. Some Knowledge-Organization MTUs which depend on the final structure of the text, are generated at this stage.

TOPIC: **third-order**
 METHOD: **general specific**
 EQUATION: **$x^3 - x^2 - x + 1 = 0$**

(ALTERNATIVE 1)
 RULE: **factor out x^2 from terms 1 and 2**
 PATTERN: **x^2 is a factor common to terms 1 and 2**
 EXPECTATION: **result has factor common with rest of terms**
 RESULT: **$x^2(x-1) - x + 1 = 0$**

RULE: **rewrite $-x+1$ as $-(x-1)$**
 RESULT: **$x^2(x-1) - (x-1) = 0$**

RULE: **factor out $x-1$**
 RESULT: **$(x-1)(x^2 - 1) = 0$**

RULE: **apply formula $a^2 - b^2 = (a+b)(a-b)$**
 RESULT: **$(x-1)^2(x+1) = 0$**

CONTINUE: **product of factors**

Fig. 1. Stylized Representation of the Technical Part of the Tutoring Strategist's Output

The Comprehension-Processes module, the Sentence Composer and the Model of the Student's Knowledge have been fully implemented, as well as those parts of the Problem Solving Expert's domain knowledge required for the text-generation task. The input to the Comprehension-Processes Module is hand coded, based on the design of the Tutoring Strategist.

A. Comprehension-Processes Module

The Comprehension-Processes Module generates MTUs for each technical message produced by the Tutoring Strategist. Each technical message is composed of a technical part accompanied by processing information.

The Comprehension-Processes Module directly derives affect-transference and most Knowledge-Organization MTUs from the structure of the technical file. Tem-

poral and additive MTUs are derived from the sequence of the rules and alternatives (see figure 1); causal MTUs are extracted from the labels of the different entities (e.g., PATTERN and RULE are translated into a construct such as “Since x^2 is a factor common to the first and second term, we factor it out”). Attributive MTUs correspond to the type of method to be discussed, namely “general” or “specific.” Finally, to generate adversative and affect-transference MTUs FIGMENT traces the expectations established by technical and meta-technical utterances, and examines the effect of forthcoming utterances on these expectations (Zukerman 1986).

Since the primary role of Knowledge-Acquisition MTUs is to prepare the listener for processing the forthcoming technical utterance, we let the Comprehension-Processes Module simulate several processes which the listener undergoes upon hearing an utterance, and use the result to identify the type of preparation required. If any of these processes results in negative affects, then a Knowledge-Acquisition MTU is prefixed to the utterance. The processes simulated correspond to the different types of Knowledge-Acquisition MTUs presented in section II.B, namely: motivational, focal, categorical, implementational and estimational. Figure 2 illustrates the process of establishing the implementation mode for the forthcoming technical utterance and generating an implementational MTU. Figure 3 depicts the process of selecting a complexity-related (estimational) MTU, and an accompanying consolatory MTU, if necessary.

Our model for selecting implementational MTUs reflects the following mental process: if a student receives a technical utterance which was already discussed, but with which he is not very familiar, he will attempt to add it to his knowledge pool (addition mode). Then, upon discovering that the accessed memory location already contains some information, he might experience disrespect or confusion. However, if

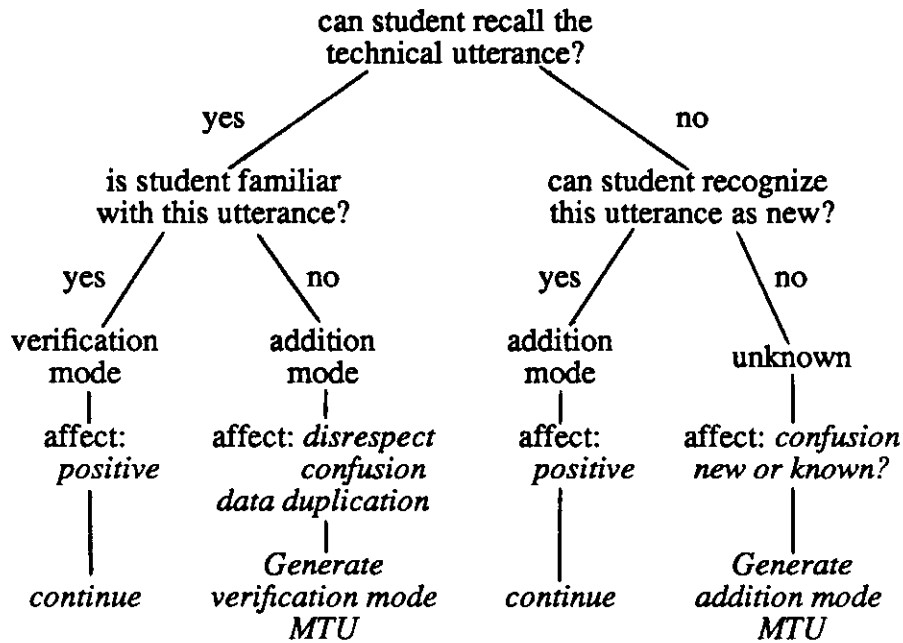


Fig. 2. Process for Generating an Implementational MTU

the tutor transfers him first to verification mode (by generating an MTU such as “As I have said before” or “Let’s go over this **once more**”), the learning process can continue unhindered. Alternatively, if a student is presented with a new technical utterance, and is unable to tell whether he has seen this utterance previously, the teacher should transfer him first to addition mode, calling for an MTU like “Let’s now consider a **new** type of equation.”

In the simplified model presented in figure 3, the difficulty of the information a student can comfortably digest depends on the complexity inherent in the technical utterance, the student’s talent and his previous mastery of this utterance. According to this model, an equation which is extremely difficult for a particular student might elicit the following text: “This equation is very difficult, however you should not be concerned, as I will go over its solution a couple of times.” A more talented student, on the other hand, may not require a complexity-related MTU for this equation.

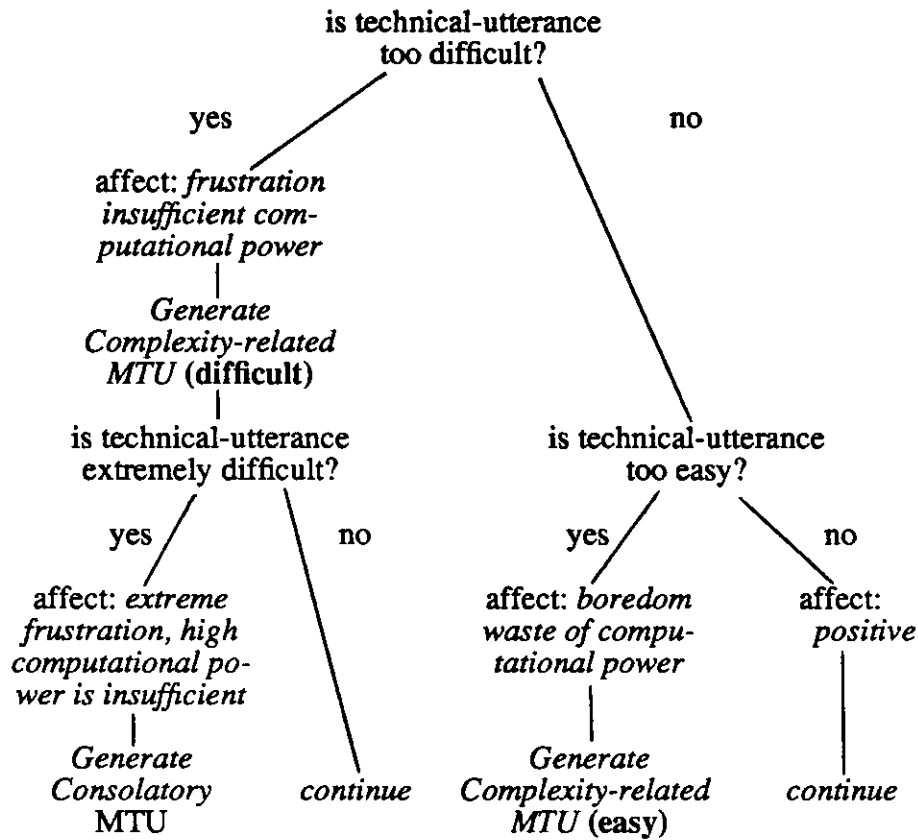


Fig. 3. Process for Generating a Complexity-related MTU

A discrimination net similar to the ones depicted above exists for each type of Knowledge-Acquisition MTU (Zukerman 1986). Each technical utterance traverses each of these nets in order to ascertain which MTUs it requires.

After the relevant MTUs have been generated, the output of the Comprehension-Processes Module is composed of a list of technical utterances interleaved with codes which specify requirements for MTUs. Table 1 depicts the MTU requirement-codes generated for the sample input in figure 1. The starred entries correspond to Knowledge-Organization and affect-transference MTUs, while the rest, correspond to Knowledge-Acquisition MTUs.

Utterance	MTU Type	MTU Code
TOPIC	<i>Motivation Focus</i>	(HIGHLIGHT ATTRIBUTES) CLOSE
METHOD	<i>*Affect Transference</i>	NEGATIVE
METHOD	<i>*Expectation</i>	VIOLATION
EQUATION	<i>Focal</i>	OPEN
RULE (Factor out)	<i>*Sequence</i>	1
EXPECTATION	<i>Implementation</i>	KNOWN
RULE (Rewrite)	<i>*Sequence</i>	2
RESULT	<i>*Expectation</i>	REALIZATION
RULE (Factor out)	<i>*Sequence</i>	3
RULE (Formula)	<i>*Sequence</i>	4

Table 1. MTU Requirement-codes for Sample Input

Most of the processes activated by the Comprehension-Processes Module rely on the hierarchical problem-solving structure of the transferred knowledge (i.e., topic, equation and solution alternatives). This structure is shared by many technical tutoring domains. The extensibility of the Comprehension-Processes Module to these domains hinges on its ability to incorporate new types of technical utterances, since the presence of a technical utterance or its accompanying MTUs may influence the need for MTUs in other technical utterances.

B. Sentence Composer

The Sentence Composer collects the technical information and the generated MTU-codes into a stylistically sound representation. To perform this task, it activates the following components.

A Phrasal Dictionary - This component applies a generation process based on the Augmented Transition Network (ATN) formalism to produce words and expressions commonly used in tutoring technical subjects. For example, the word “new,” or a sentence like “we have never seen this topic before” may be generated from the following dictionary entry:

NEW = {“new,” “\$person have (never) (study 1p pp) \$subject (before)”}

An Attribute-clause Generator - This component produces sentences containing attributes of a given item, and information regarding the knowledge-status of the student with respect to this item. For instance, the following sentence is generated by this component: “We shall consider a very important topic, which we have not encountered for a while, and is quite challenging.” In order to generate this type of sentence, the attribute-clause generator applies rhetorical rules, which determine the number of clauses to be produced, and collect the attributes into clauses.

Utterance Generators - The English representation of an MTU and the manner in which multiple MTUs interact depend on the technical utterance providing the context. Therefore, for each type of technical-utterance (e.g., topic, method, pattern, etc), the Sentence Composer features a dedicated text generator, which applies rhetorical rules to determine the order and manner in which technical and meta-technical utterances shall be presented. These generators enable FIGMENT to generate several MTUs that perform the same function (e.g., specifying implementation mode), but

whose English representations differ according to the type of the technical utterance under consideration. For instance, the MTUs “As I have stated before” and “This equation is similar to ...” put a student in verification mode; however, while the former refers to explanations, the latter applies to equations.

The following text illustrates a typical output of FIGMENT’s Sentence Composer:

1 **Let us now look at a rather interesting topic,**
2 **namely third degree equations, which is also**
3 **challenging. Unfortunately, we shall not examine**
4 **a general technique for solving equations in this**
5 **subject. However, we can solve certain types of**
6 **third degree equations by factoring out common**
7 **factors, or, alternatively, applying the appropriate**
8 **factorization formula. Here is an equation:**
9 $x^3 - x^2 - x + 1 = 0$
10 **First, since x^2 is a factor common to the first and**
11 **second terms, we factor it out. As you know, we**
12 **perform this operation hoping to get a factor**
13 **common to the rest of the terms. Through it we**
14 **get the following result:**
15 $x^2(x-1) - x + 1 = 0$
16 **Next, we rewrite $-x+1$ as $-(x-1)$, arriving at the**
17 **result we were hoping for:**
18 $x^2(x-1) - (x-1) = 0$
19 **Afterwards we factor out $x-1$, yielding:**
20 $(x-1)(x^2 - 1) = 0$
21 **We continue by applying the factorization**
22 **formula $a^2 - b^2 = (a+b)(a-b)$ to x^2-1 , arriving at**
23 **the following result:**
24 $(x-1)^2(x+1) = 0$
25 **We obtain the solution by solving separately for**
26 **each factor.**

IV CONCLUSIONS

It is generally believed that any system which generates continuous discourse must contain models of both the process by which a listener absorbs information and the affective impact of this information. This paper offers a concrete design of the makeup of these models and their incorporation in a text generation system as tools

for generating fluent and cogent text. Specifically, this paper presents a generally applicable operational taxonomy of MTUs, and demonstrates its usefulness in maintaining continuity in multi-sentential text. It also shows the sufficiency of shallow models of the listener Comprehension-Process to weave appropriate MTUs into technical discourse. The text generated by using these models captures sufficient rhetorical features to support continuous discourse.

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