CONVINCE: A CONVERSATIONAL INFERENCE CONSOLIDATION ENGINE

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by

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1983

To my wife

Soon Jung Kim

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VITA

March 27, 1949 -- Born, Seoul, Korea

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ABSTRACT OF THE DISSERTATION

CONVINCE: A CONVersational INference Consolidation Engine

by

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Doctor of Philosophy in Computer Science

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An interactive domain-independent decision support system has been designed and implemented. The system assists a human decision-maker in situation assessment tasks through a cyclic process of information search and integration. The system elicits the user's perception of a given situation in an English-like dialogue, focusing the user's attention on the issues of highest relevancy. Elicited problems are structured as hierarchical networks where nodes represent variables and links represent causal relationships. Each causality link is quantified by a conditional probability matrix specifying the probability of each possible effect, given its causal factors.

The system uses a Bayesian inference procedure which is a generalization of previous methods applied to tree, in that it permits the modeling of multiple causes to a given manifestation. The inference technique synergistically combines causal and diagnostic reasoning using a bi-directional propagation of evidence through the network. Belief parameters and propagation formulas are established which permit all belief parameters to be updated in a single pass through the network with the arrival of each new piece of evidence.

Upon completion of the dialogue, the system provides a formal structure representing the relationships between the relevant variables, and their updated belief distributions.

Chapter 1

INTRODUCTION

1.1 THE TASK

Situation assessment constitutes a fundamental problem in many decision-making processes such as medical diagnosis, military and business planning. The task of situation assessment can be viewed as a cyclic process of information search and integration. The decision-maker starts with some uncertainty with regard to some true situation, and then looks for additional information which may reduce this uncertainty. The new information is integrated into the existing information and the situation is reassessed. If a final assessment cannot be made, further information is requested. The process ends when the decision-maker decides either that he knows enough about the situation and can make up his mind, or that no additional sources of information can contribute significantly to remove the remaining uncertainties.

This thesis demonstrates a domain independent interactive decision support system for situation assessment. The system, called CONVINCE (CONVersational INference Consolidation Engine), is designed to assist, not replace, a human decision-maker both in planning a search for information and data, and in the rational integration of the gathered evidence.

CONVINCE is a problem-structuring system which helps a user articulate ill-defined problems in a formal structure, and which then deduces rational inferences from the formal structure. This type of system may be viewed as an automated knowledge acquisition tool for constructing coarse-grain knowledge-based systems; the skeleton belief structure elicited can be used for other problems when similar situations arise.

This thesis also describes a Bayesian inference procedure which was devised for and is used by CONVINCE to model both causal and diagnostic modes of reasoning simultaneously. The causal mode of reasoning refers to the process of updating the likelihood of an event due to modified belief in its causal factors, while the diagnostic refers to that of updating the likelihood of an event as a result of an update in some of its manifestations[Tversky and Kahneman 79, Burns and Pearl 82]. The inference procedure devised is efficient in the sense that the beliefs of related variables are updated by local computations in a single pass, avoiding infinite relaxations.

1.2 EXPERT SYSTEMS VERSUS PROBLEM STRUCTURING SYSTEMS

Computer-based decision aiding (or decision support) systems can be classified into two major categories: knowledge-based expert systems and situation-based problem structuring systems. The former maintain a large data base which contains domain specific factual information as well as heuristic inference rules applicable to a narrow problem domain, these provide recommendations based on the rules and facts stored in the data base. The later are domain-independent, acquiring knowledge and generating inferences concurrently. Most of the background knowledge and expertise are carried by the user himself, not by the system; the system's main role is to provide a skeletal structure in which relevant concepts and facts can be represented. Recommendations are then derived according to prescribed normative inference rules. In this kind of decision aiding system, the computer acts as a sophisticated and friendly "sounding board". It does not provide information of its own, but first assists the user to search, articulate, and structure his own knowledge and then generates conclusions mechanically drawn from that knowledge.

Whereas expert systems are intended to replicate human expert in a high technological domain, problem structuring systems work cooperatively with the human decision-maker. This reflects a paradigmatic difference between the two fields in which these systems have evolved. Artificial Intelligence(AI), which developed expert systems, aims at making machine cognition similar that of human, while Decison Anaysis(DA) emphasizes the difference between the two and aims at having them complementarily.

The utility of knowledge-based systems versus problem structuring systems depends mainly upon the repeatablity of the decision task under study. Knowledge-based systems are, in general, more effective in supporting repeatable decision tasks because they provide a deeper analysis. In unique decision tasks, however, the time it takes to construct a knowledge base cannot be justified and situation-based systems are more attractive.

Situation-based problem structuring systems can also be used as automated knowledge acquisition tools for constructing knowledge-based systems. The same skeletal structure elicited and formalized during an interview with domain experts can be used as a source of knowldge and data when similar situations arise. Although intelligent expert systems require a deeper analysis and a larger amount of domain specific knowledge in a more diverse and elaborated format than that employed by situation-based systems, a shallow but quick analysis may be useful to identify crucial areas and to obtain suggestions on the directions of further search for knowledge.

1.3 HISTORICAL REVIEW OF SITUATION-BASED PROBLEM STRUCTURING SYSTEMS

Decision analysts are often called upon to assist a decision-maker in the solution of complex and critical decision problems. The major contributions of these experts lie mainly in their ability to cast a problem into a formal structure from which inferences can be deduced mechanically. Even though the decision analyst possesses less domain-specific knowledge than the decision-maker, the inferences deduced from the formal structure will often prove to be more accurate and reliable than those deduced by the unaided decision-maker.

Decision analysis is a general-purpose decision aiding technology which has been successfully applied to a wide variety of problem domains for the last 2Ø years[Raiffa 7Ø, Howard 76, Brown 74]. Central to the technique is the decomposition of a decision problem into actions, events and consequences, to which likelihoods and preference relationships are then assigned. Normative rules and logic are then applied to select the preferred course of action(s). This technique is founded on the paradigm that although people possess reliable procedures for acquiring, storing and retrieving fragments of knowledge, they possess much less reliable procedures for combining these fragments into a global inference. Thus, it is desirable to augment human decision-making process with mechanical inference algorithms for combining knowledge fragments.

Recently, the use of computers has been applied not only to the task of selecting an optimal course of action, but also to the structuring of a decision problem as perceived by a decision-maker. These computer systems, called decision structuring aids or, more generally, decision support systems, differ from 'informational aid' systems in that the latter only help in organizing, computing and displaying data without directly addressing a decision process itself. Previously, several computer aids became available for use in eliciting isolated elements of information required for a decision process. These include programs to encode subjective probability distributions[Spetzler and Holstein 75], elicit multi-attribute utility functions[Keeney and Sicherman 75], and so on. However, unlike decision structuring systems, these programs were not intended to deal with the entire process of decision-making from problem structuring and formulation to alternative evaluation and the selection of choices.

Computerized decision structuring systems provide three distinct advantages. First is their capability to conduct real time sensitivity analysis, which makes it possible to to guide the growth of the structure in only the most promising directions. By focusing the user's attention to the area most crucial to the main decision related goals, a more reliable conclusion would be derived, even with a simpler structure. Second advantage is the ease with which they permit a system to be updated with new knowledge. Elicited structures are saved and reused by adding new knowledge to the old structure. Third is provision of a common ground on which a group of experts may cooperate in the solution of a large, complex problem. Since different experts may perceive different aspects of the same problem, disagreements could be detected, isolated and brought up for further discussion.

Full-scale decision structuring systems usually operate in either user-initiated or system-guided modes. Systems operating in the former mode provide the decision-maker with a language in which he may describe the components of the problem and the relationships among them. Systems operating in the later mode conduct a question-answering interview (in stylized English) and elicit the user's perceived structure in a systematic fashion. Full-scale decision structuring systems also derive conclusions applying predefined logics on the elicited structure and, in addition, can identify critical decision variables and focus the user's attention on the area of the highest relevancy.

A full-scale decision structuring system based on a decision tree formulation is found in Leal and Pearl[Leal 77]. This system starts the interview by asking the decision-maker to identify alternative actions available at the main decision point. Each action's consequences, together with their likelihood and preference measures, are then elicited. By iterative application of this process, a decision tree is constructed incrementally to capture the essence of the decision problem. This system performs an alternative analysis and selects a recommendation based on the criteria of the maximum expected utility. It is also equipped with a mechanism for controlling the focus of attention.

Another system in this category, called GODDESS, has been developed by Pearl, Leal and Saleh[Pearl 82a]. Departing from the traditional decision-tree approach, GODDESS utilizes a formalism borrowed from AI robot-planning applications where goals, actions, conditions and events constitute the basic building blocks, and the relationships between these entities are assessed by the user. The system starts by focusing attention on the goals to be achieved and outcomes to be avoided. Then it leads the decision-maker first to the identification of the means by which these objectives can be realized, then to the detailed pre-conditions which need to be prepared in order to make them effective. Pearl, Leal and Saleh claim that this bootstrapping structuring method helps guide the user toward the discovery of action alternatives he otherwise may not identify. GODDESS also uses a discourse management algorithm, computing the expected value of analysis, to guide the dialogue along paths which will gain the most benefit from further exploration.

The system developed by Merkhofer and Leaf[Merkhofer 80] at SRI also belongs to this category. The structuring process of this system consists of three phases: preliminary structuring, modelling, and expansion. In the prelimilary phase, the basic factors such as decision objectives, alternatives and critical uncertainties are extracted. These factors are organized into the decision-tree structure in the modelling phase through analyses on a network called an influence diagram[Owen 78]. This phase of analysis provides insights and assistance in the identification of tentative areas of decision strategy. The expansion phase refines the analysis of those areas of the model to which the decision is most sensitive. Display graphics are used for reviewing the elicited structure, and English queries for the extraction of decision attributes.

A system by Henrion[Henrion 79] casts acquired knowledge in the form of equations. A decision-maker starts the dialogue by first defining a goal variable and then specifying that by an equation. The system detects undefined variables and prompts for information in order to define these variables. The needed information includes such components as value distribution, English descriptions, units, justification, and whether they are exogenous or not. The data structure for variable representation is similar to the frame-based representation scheme of AI systems. If all exogenous variables are bound, either by a specific value or a distribution, the system computes the value or distribution for the target variable. This system's main focus is on policy decisions in which uncertainties exist and a large amount of quantitative analysis is appropriate.

Knowledge acquisition systems, whose primary function is to transfer of the knowledge of human experts to expert systems, can also be useful for structuring both problems and problem-solving tactics. These systems usually adopt a formal representation scheme and prompt the user for the information necessary to fill the slots of the scheme.

MYCIN was the first expert system in which a limited knowledge acquisition tool was provided for use by a knowledge engineer to modify the contents of the knowledge-base[Shortliffe 76]. MYCIN's basic knowledge representation scheme is a production rule, cast in the form of premise-consequence pair with an associated certainty measure. The system understands rules expressed in stylized English, and also writes back what it understands for confirmation by the knowledge engineer.

An interactive knowledge acquisition system for rule-based systems developed by Davis called TEIRESIAS uses a schema hierarchy that contains knowledge about the representation of knowledge[Davis 78]. The elicitation of knowledge is performed by a top-down interpretation of the meta-knowledge schema. The user is prompted to supply the missing parts of the required knowledge during the interpretation. The system can understand a subset of the English language: syntactic checking as well as semantic checking is performed to maintain data-base consistency. The system can also step through an inference chain in a debugging mode, allowing the 'knowledge supplier' to intervene at any point to modify, delete or add production rules.

EMYCIN is a domain-independent system for constructing rule-based systems[van Melle 80]. As its name implies, its basic structure is the same as that of the MYCIN. Here, guided by the system, a 'knowledge supplier' provides the information needed to construct context trees that represent knowledge organization hierarchies. Rules are then defined using the terminologies of the context trees. The system supports both a debugging procedure to provide flexibility and a rule compiler for achieving efficiency.

Duda, Hart, Konolige and Reboh[Duda 79] describe a knowledge acquisition system called KAS for the PROSPECTOR system. PROSPECTOR employs an inference network for expressing taxonomic and other static knowledge. These networks may be constructed either by a system-guided interview or a network editor. The network editor understands the commands for the creation, modification and the deletion of network elements. Like TEIRESIAS, a controlled execution is allowed in order to validate the correctness of new knowledge.

All these systems represent both the user's perception and knowledge about problems in formal structures. Conclusions are derived from those by applying prescribed rules of decision-making. As discussed earlier, such formal structures provide a common ground for further analysis and communication among experts. The systematic guidance employed by some of these systems advises the decision-maker in search of information and brings more alternative action strategies into consideration through focusing attention on a small area. Their interactive nature and capacity for real-time computation enhances their utility as a real-time decision-making aid. A friendly and graceful human interface through a natural-language interface and/or a graphics system would further enhance their effectiveness.

1.4 UNIQUE FEATURES OF CONVINCE

CONVINCE explicates the user's perception of relevant variables and their causal relations from an interview conducted in an English-like dialogue. It aims to help the user by structuring his problem and deriving conclusions through a logical analysis. Unlike several decision structuring systems that emphasize the selection of optimal actions, CONVINCE emphasizes the situation assessment aspect of the decision making process, i.e., the assessment of the likelihood of relevant events. The reason that it focuses on this aspect is because selection of optimal actions often becomes trivial once precise assessements of uncertain events are obtained. Also, many real world problems including diagnosis, data interpretation, forecasting and prediction fall into this generic category.

Unlike knowledge-based decision support systems, CONVINCE can generate recommendations in a relatively short time (few hours), avoiding the time consuming knowledge-base construction phase that usually stretches over several months or sometimes even years. CONVINCE aims to help decision-makers who are facing unique, non-recurring problems that demand quick responses. CONVINCE achieves this goal by generating reasonable recommendations through a shallow, but quick analysis concurrently acquiring domain knowledge.

In contrast to most AI knowledge-based systems, CONVINCE's inference scheme is based on formal probability calculus. This scheme facilitates the representation of multiple causes in a natural way and is still computationally efficient. More importantly, it is compatible with human reasoning.

CONVINCE is user-friendly in the sense that it queries only what the user can easily answer. For example, although CONVINCE utilizes both causal and diagnostic information, it asks the user to quantify relationships only in the casual direction, i.e., in the form of probability of a manifestation given a cause. Elicitation in that direction is more compatible with formats used by people to model invariant aspect of their environment. Consequently this elicitation mode is more natural and, therefore, easier than that of the anti-causal direction, and more likely to produce valid and consistent judgment. Natural language dialogue and intelligent control of the user's attention are other graceful features designed for a quick but faithful represeatation of the user's perception.

1.5 OVERVIEW OF THIS DISSERTATION

Chapter 2 describes the problem representation formalism employed by CONVINCE and the assumptions implicit in its structure. Also discussed are psychological issues such as the validity and consistency of subjective probability assessments. The inference algorithm is described in Chapter 3 after a brief review of other approaches to the inference problem. Chapter 4 describes the overall structure of CONVINCE and the details of its component modules. A prototype of CONVINCE is demonstrated in Chapter 5 where a typical dialogue with a decision-maker facing a hypothetical problem is shown. Finally, Chapter 6 discusses the assumptions and limitations inherent in the CONVINCE approach. Further directions of research and development are identified.

1.6 IMPLEMENTATION NOTES

The currrent implementation of the prototype of CONVINCE is written in INTERLISP and runs on a DEC PDP-2Ø under TOPS-2Ø operating system.

Chapter 2

CAUSAL NETWORKS

2.1 NETWORK ELEMENTS

Whenever a person expresses a quantified belief either numerically or linguistically, it is the result of a mental summarization of a semantic network in which relevant concepts, facts and their relations are encoded. This cognitive process is not always rational and consistent. Explicating relevant variables and their relationships provides a ground on which normative rules of decision-making might be applied. Explicating human cognitive structures and processes constitutes the main task in computer-aided problem solving.

We postulate that people perceive variables and relations relevant to making inferences in the form of a network of causal relationships. In this network, refered to as a causal network, each node represents a variable and each link represents a causal relationship between two variables. Each variable represents a finite partition of the world given by the variable values or states. It may be a name of a collection of hypotheses(e.g., Identity of Organism: ORG1, ORG2) or a collection of possible observations(e.g., Patient's temperature: High, Low, Medium). We shall denote variables by capital letters, e.g., A, B, C, and subscript their various states by numbers such as A_1, A_2.

A causal network is a directed graph where each link X ----> Y represents the cause-effect relatioship 'X causes Y'. Three commonly accepted conditions must hold to claim that X causes Y[Kenny 79]. These are:

1. Time precedence

2. Dependency

3. Nonspuriousness

For X to cause Y, X must precede Y in time, thus rendering the causality relationship asymmetric. The second condition for causation is the presence of a functional dependency or correlation between the variables. The third condition prevents us from misinterpreting the relationship between X and Y as a causality when a third variable, Z, causes both X and Y. We call the relation between X and Y spurious if the statistical dependency between X and Y vanishes once Z is controlled. A distinction should also be made between a spurious relation and relations through intervening variables. A variable W intervenes between X and Y if X causes W and W in turn causes Y. See Figure 2-1.

Each node[1] in a causal network is characterized by a set of mutually execlusive and exhaustive states, each with its own probability, or, belief, of occurence. The relationship X ----> Y is quantified by a conditional probability matrix $\underline{M}(Y|X)$ with entries:

$$(2\text{-}1) \quad \underline{M}(Y|X)_{i,j} = \mathrm{Prob}(Y_i|X_j).$$

The directionality of the arrow designates X as the set of hypotheses and Y as their set of indicators or manifestations. We restrict the arrows to follow the direction of causality. In other words, relations among variables are characterized by conditional probalibilities where the cause, not the effect, is the conditioned variable.

The validity of conditional probability assessments P(X|Y) of a target event X on the basis of some evidence Y has been the subject of recent studies[Tversky and kahneman 77, Burns and Pearl 79, Burns and Pearl 82, Moskowitz and

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1. In further discussions, we will use the terms 'node' and 'variable' interchangably.

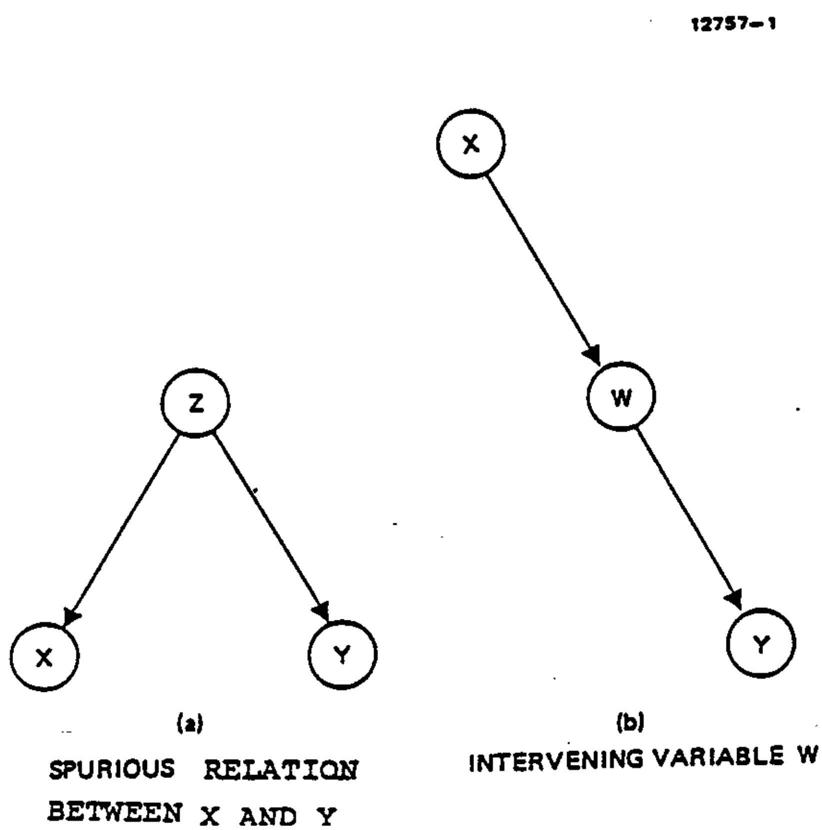

Figure 2-1: Spurious RELATION and Intervening Variable

Sarin 81]. Tversky found that people perceive the impact of one event on another differently depending upon the causal relationship between them. With his terminology, the relationship between X and Y is causal if Y is perceived as a cause of the occurence or nonoccurrence of X. Conversely, the relationship is diagnostic if Y is perceived as a manifestation of X. Tversky found that a causal relation is perceived to be more informative than its diagnostic counterpart, but reserved the conclusion as to which mode of reasoning is more valid. He added, though, that people find the causal mode of reasoning easier, more natural and more confident than the diagnostic mode. A recent experimental study showed no significant difference of validity between the two modes of reasoning[Burns and Pearl 82]. However, because this study involved judgments about facts for which subjects had only sketchy knowledge or data, its findings may not be applicable in situations involving expert judgments. The rest of this dissertation is based on the assumption that in many occasions the probability P(manifestation|cause) is more available and, therefore, can be elicited with greater ease and validity than its counterpart P(cause|manifestations). Although Bayes' rule allows us to derive P(effect|cause) from P(cause|effect) if the prior probability P(cause) is available, we prefer to assess the former directly and infer the latter mechanically (by the Bayes' rule).

Causal relations form a hierarchy in which one variable may play the role as a causal factor for a set of variables, and, at the same time, represent a manifestation of another set of variables. The relation between the target hypothesis and the observed data is represented as a cascade of a local probability relations involving intervening variables. Intervening variables may or may not be directly observable. Their computational role in an information system, however, is to provide a summarization for loosely coupled subsets of the observational data so that inference computation can be performed by local processes, each employing a relatively small number of data sources.

In order to present a more detailed discussion of the network, we need to introduce some definitions (see Figure 2-2.).

- A path from one node to another is a set of causality links connected head to tail that forms a directed line from one node to another.

- An underlying path from one node to another is a set of causality links connected regardless of their direction.

- A node A is a predecessor of node B if there exists a

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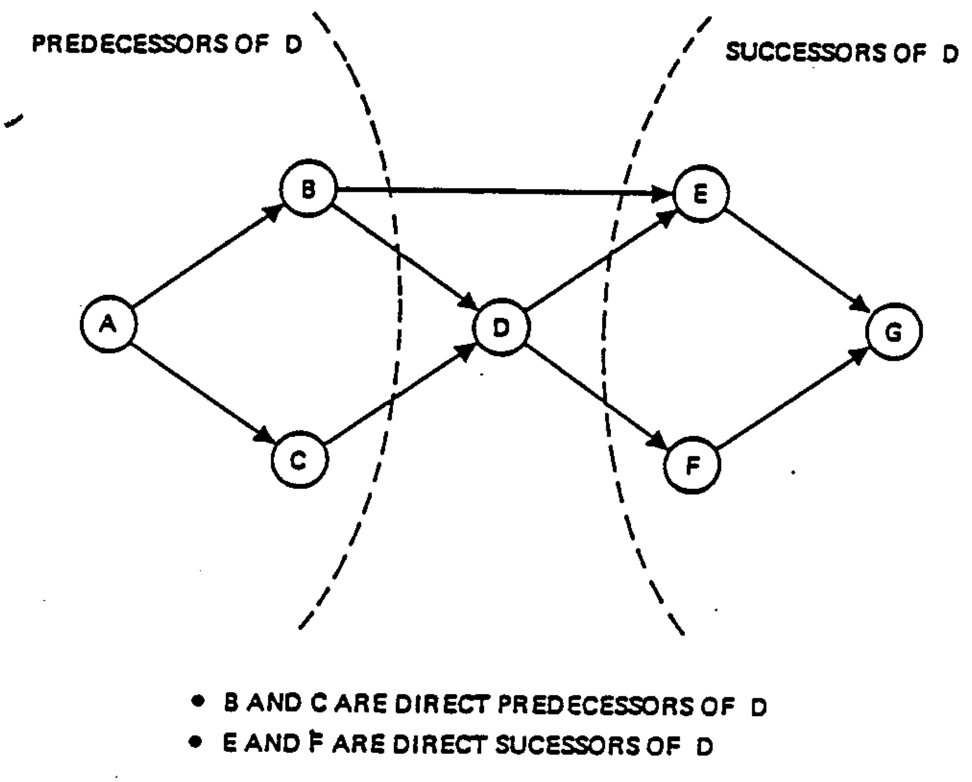

Figure 2-2: Predecessor/Successor Relationship

path from A to B.

- A node A is a direct predecessor of node B if there exists a causality link from A to B.

- A node A is a successor of node B if there exists a path from B to A.

- A node A is a direct successor of node B if there exists a causality link from B to A.

We divided the nodes into three different types according to the roles of their associated variables in the information system.

- A target node is a node corresponding to the variable of direct interest.

- A data node is a variable whose state is observed, or may be observed, with certainty.

- An intervening node is a node which is needed to tie the target node to a data node.

We may place the intervening nodes into two categories depending on the amount of information available at any given moment when the network is constructed incrementally.

- An expanded node is an intervening node which has been fully explored, i.e., its relationships with all its neighbors are fully explicated.

- An unexpanded node is an intervening node which has not been fully explored yet.

The representation of the causal network has a limitation. The representation framework can accommodate only the information provided by second order[2] probability distributions. For example, a general third order probability distribution P(A,B,C) cannot be specified in the representation of the causal network. The best way a model builder can deal with a third order probability distribution is to approximate it by pairwise relationships, such as P(A|B), P(C|B), and P(B|C). This, however, is not a significant drawback because high order probability distributions are generally unavailable, and they are unreliable even if they are assessed.

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2. We use the term "order of probability distribution" to signify the number of distinct variables describing the probability distribution. For example, the order of P(A) is one, order of P(X,Y) and P(X|Y) is two.

As an illustration of the representation scheme for the causal network, consider the following situation:

> Mr. Holmes received a telephone call from his neighbor notifying him that she heard a burglar alarm sound from the direction of his home. As he was preparing to rush home, Mr. Holmes recalled that last time the alarm had been triggered by an earthquake. On his way driving home, he heard a radio newscast reporting an earthquake 200 miles away.

The causal network representing Mr. Holmes' belief structure is presented in Figure 2-3. The concentric circles represent the target node(BURGLARY), other circles represent intervening nodes (i.e., variables whose states are uncertain), and triangles represent data nodes (i.e., variables whose prevailing state is observed and known with certainty). We will restrict our attention to a special kind of graph referred to as a Chow Tree or a singly connected graph, in that no underlying cycle[3] exists. In a Chow Tree, although a node may have multiple parents, there exists only one underlying path between any pair of nodes in the connected tree. If any of the links are deleted, the Chow Tree forms two disjointed subgraphs. In general, if the perceived inference network contains cycles, it cannot be represented in the form of a Chow Tree. This makes for a less efficient and tractable computation. Therefore, to comply with the requirements of the hierarchical

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3. This means there is no undirected cycle in the underlying, undirected graph.

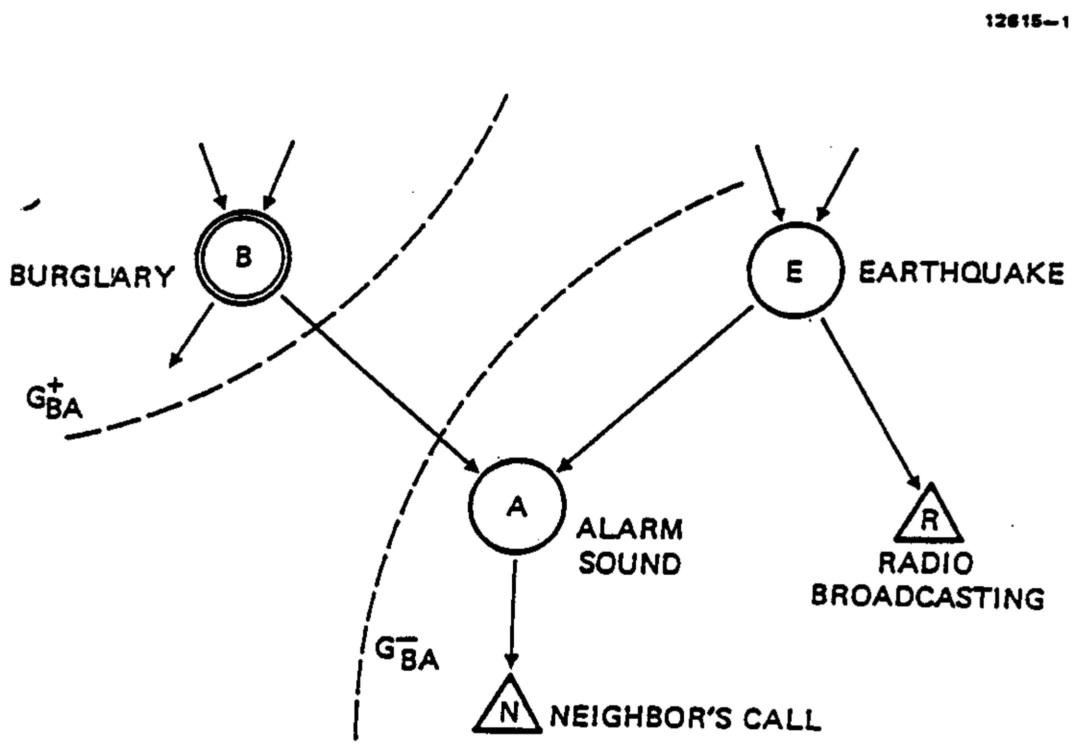


Figure 2-3: Mr. Holems' Belief Structure

representation adopted by CONVINCE's inference engine, a method has been devised by which a general causal network with cycles is converted into a Chow Tree through systematic approximations based on information theoretic considerations.

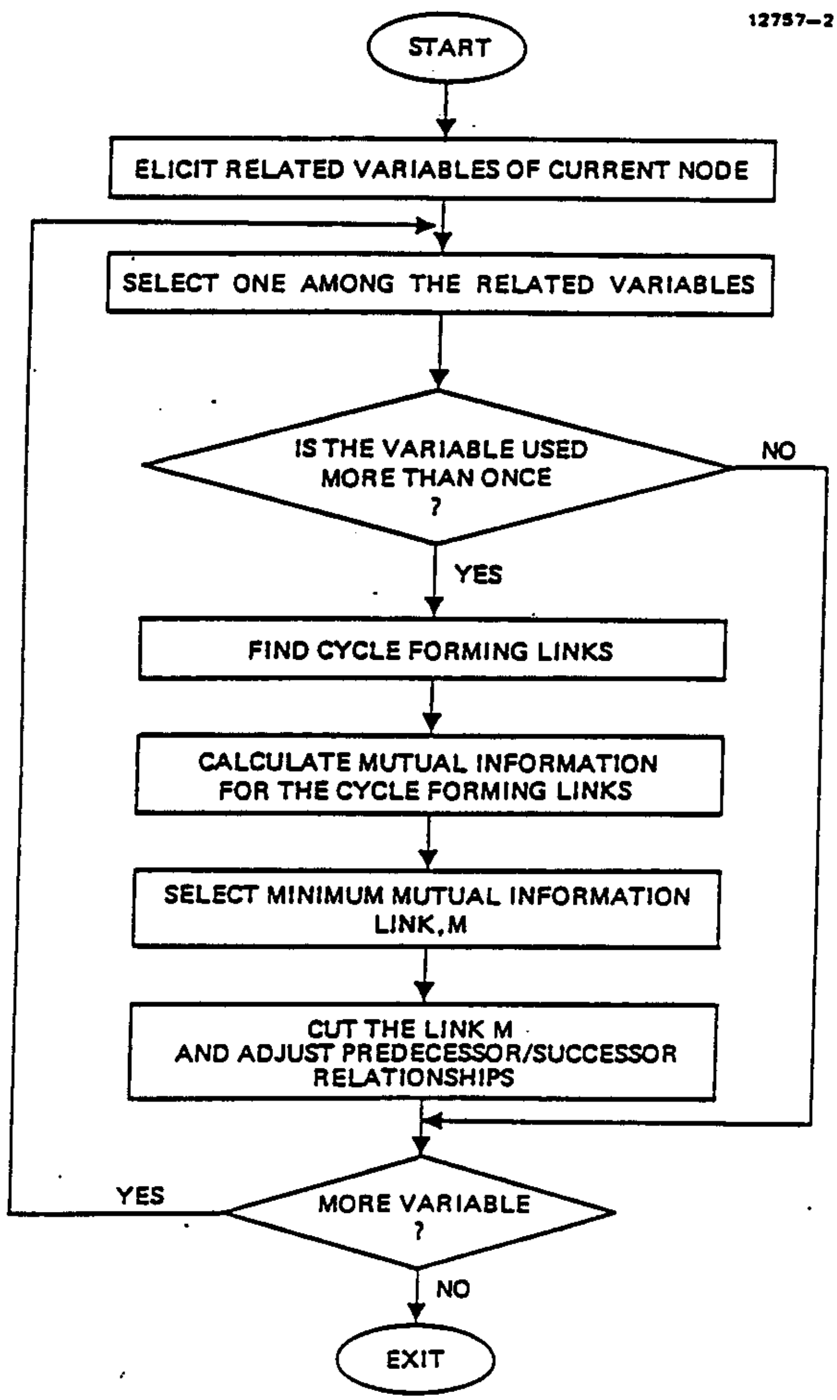
This approximation is based on the works of Chow and Liu [Chow 68] who devised a method for approximating optimally N-dimensional discrete probability distributions by a product of second-order, pairwise distributions. The approximation is optimal in the sense that the approximated distribution preserves maximal information among distributions that can be approximated by a product of N-1 second-order conditional distributions. They showed that the optimal approximation corresponds to the maximum spanning tree of a graph which is formed by representing variables as nodes and pair-wise relationships as links, then assigning the links by the mutual information of the two variables located at each end of a link. Further details of their work is included in Appendix I.

CONVINCE uses Chow and Liu's approach of extracting the most informative tree from a general causal network. However, the extraction process is required to be modified to work in the CONVINCE's incremental mode of network construction where newly acquired information is added to the existing network. In this mode of construction, a high dependency link may become available after a weaker dependency link is processed. Therefore, at each step of growing network, CONVINCE tests whether cycles have been formed and if that is the case, it deletes the least informative link from that cycle. Note that formation of a cycle is easily detected by checking whether the description of a node is used more than once. The flow of the algorithm that incrementally constructs a Chow Tree is given Figure 2-4.

2.2 STRUCTURAL ASSUMPTIONS OF INDEPENDENCE

We have mentioned that we will restrict our attention to a special kind of graph called the Chow Tree where although a node _may_ have several parents at most one underlying path exists between any pair of nodes. Since no cycles exist, a link B --> A partitions the graph into two parts: an upper subgraph, G^{+}_{BA}, and a lower subgraph, G^{-}_{BA}. These two graphs constitute hierarchical representations for the sets of _data_ that impinge on these graph. The sets of data will be called as D^{+}_{BA} and D^{-}_{BA}, respectively. These data are defined by the observations and prior beliefs obtained at the _boundaries_ of a network. Likewise, any node A partitions the graph into two parts: above A, G^{+}_{A}, and

Figure 2-4: Algorithm for Maintaining Acyclic Network



below A, G^-_A, representing the data sets D^+_A and D^-_A, respectively (See Figures 2-3 and 2-5). We call the variables included in G^+_A causally influencing A and those in G^-_A diagnostically influencing A. Note that for each causally influencing variable of A there is a unique underlying path from that variable to a direct predecessor of A; for each diagnostically influencing variable of A there is a unique underlying path from that variable to a direct successor of A.

When we interpret the relationships among variables in a Chow Tree, we introduce three assumptions depending upon the causality relationships among them: cross-generation independence, inter-symptom independence and inter-cause independence. Cross-generation independence characterizes the relationship between a node and its grand descendants[4] or ancestors[5]. Inter-symptom independece characterizes the relationship among manifestation variables which share a common cause. Inter-cause independence characterizes the relationships among causal variables which share a common manifestation variable.

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4. successors of successors.

5. predecessors of predecessors.

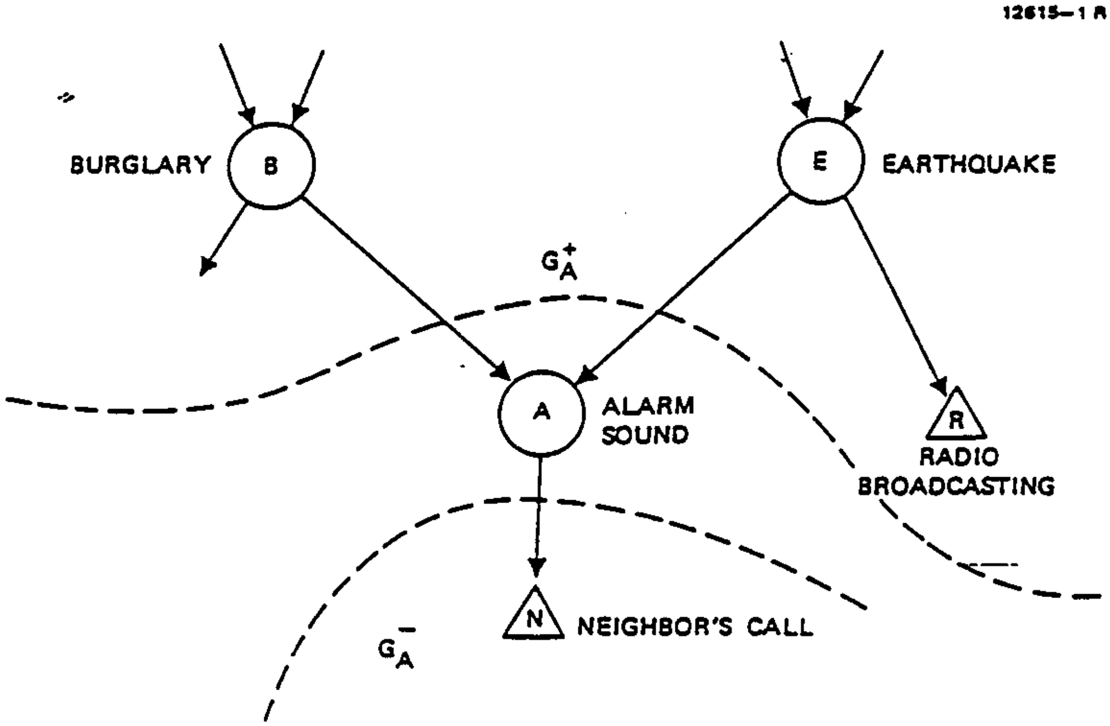


Figure 2-5: Causally Influencing Variable and Diagnostically Influencing Variable.

Cross-Generation Independence

Cross-generation independece states that the influence between a grandparent and a grandson is completely summarized by an intermediate node between them. Consider the fragment of a causal network in Figure 2-6(b). The data above B, D^{+}_{B}, influences X only through the states of A:

$$(2\text{-}2) \qquad P(X_i | A_j, D^{+}_{B}) = P(X_i | A_j)$$

which leads to:

$$(2\text{-}3) \qquad P(X_i | D^{+}_{B}) = \sum_j P(X_i | A_j)\, P(A_j | D^{+}_{B}).$$

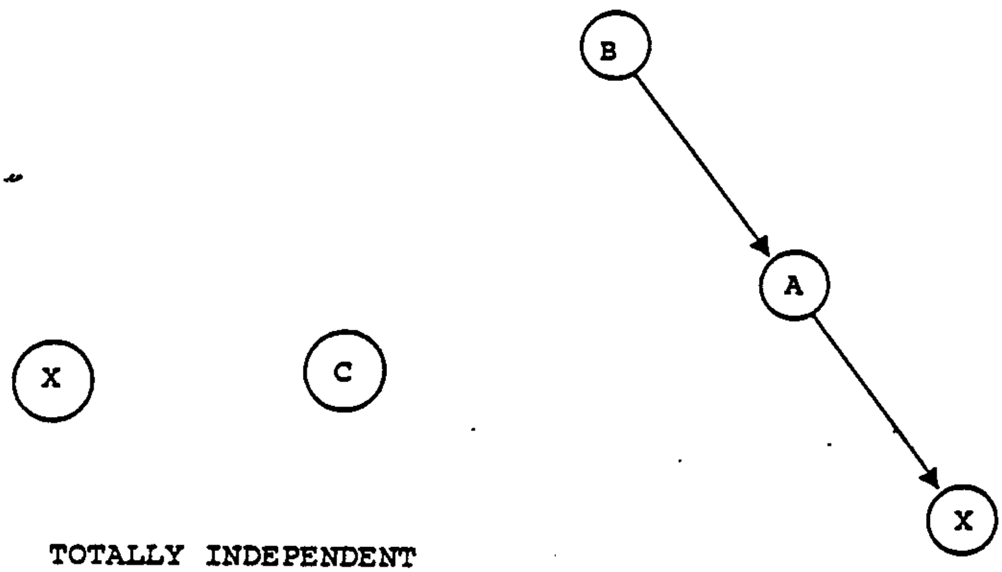
In classical terms, the states of the direct predecessor nodes are sufficient statistics for indirect predecessor/successor relationships. This assumption yields the following formula for combining influences of both a predecessor B and a successor X on a given node A:

$$\begin{aligned}(2\text{-}4) \qquad P(A_j | X, B) &= P(X | A_j, B)\, P(A_j | B) \,/\, P(X | B) \\ &= \beta\, P(X | A_j)\, P(A_j | B)\end{aligned}$$

where β is a normalizing constant such that:

$$\begin{aligned}(2\text{-}5) \qquad \beta &= 1 / P(X | B) \\ &= 1 / \sum_j P(X | A_j, B)\, P(A_j | B).\end{aligned}$$

Note that the cross-generation independence assumption is equivalent to what is traditionally called a "Markov



TOTALLY INDEPENDENT VARIABLES X AND C

(A)

CROSS-GENERATION INDEPENDENT VARIABLES B AND X

(B)

Figure 2-6 : Independence Relations (I)

assumption", where a node is independent of its grandfather once its father becomes known.

Inter-Symptom Independence

Inter-symptom independence is a property of the relationships among several manifestations of a common cause. If X and Z are successors of Y (see Figure 2-6(c)), we then assume that:

$$P(X_i, Z_j | Y_k) = P(X_i | Y_k)\, P(Z_j | Y_k) \tag{2-6}$$

which means that X and Z are not independent *a priori*, but become independent once we know with certainty which state of Y prevails. This assumption is equivalent to what is traditionally called "conditional independence". It is usually valid among several manifestations of a common cause but not among the causes of a given manifestation.

Inter-Cause Independence

Inter-cause relation is typically perceived to work in the opposite direction to the inter-symptom relation where causes are viewed to be a priori independent but once their common symptom is observed they become coupled. In Mr.

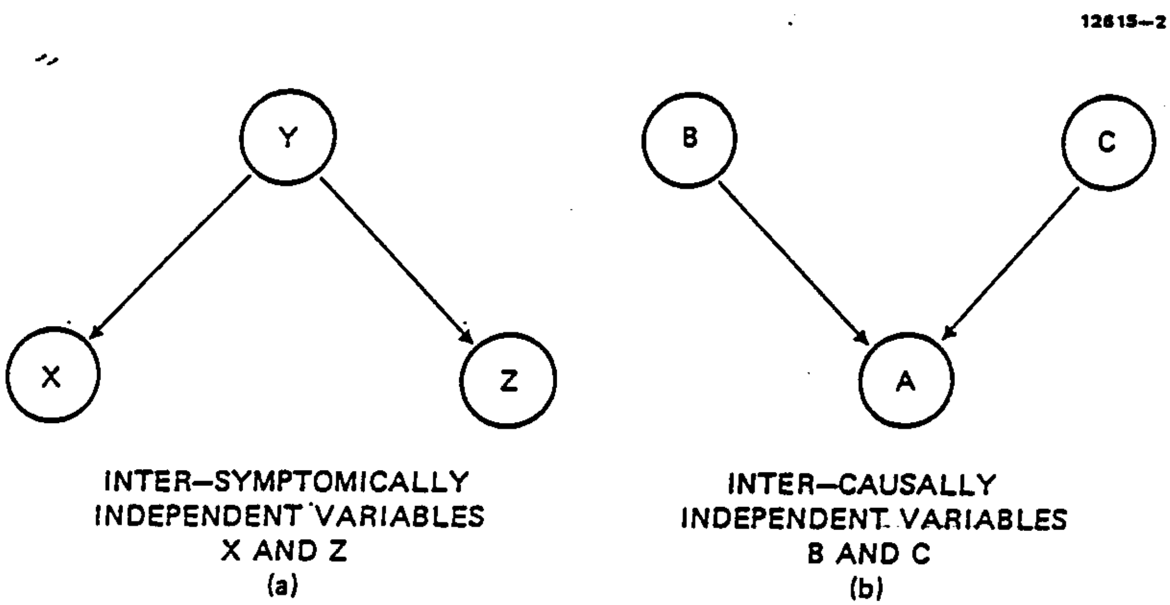


Figure 2-7: Independence Relations (II)

Holmes example, burglaries can safely be assumed to be independent of earthquakes. However, given the alarm sound, the likelihood of a burglary becomes dependent upon the occurrence of an earthquake. We call this relationship inter-cause independence, and formulate it :

$$(2\text{-}7) \qquad P(B_i, C_j | D^+_B, D^+_C) = P(B_i | D^+_B)\ P(C_j | D^+_C),$$

but

$$(2\text{-}8) \qquad P(B_i, C_j | A_k) \neq P(B_i | A_k)\ P(C_j | A_k)$$

where A_k is a state of a common diagnostic variable (see Figure 2-6 (d)).

The inter-cause and inter-symptom independences have the following graph interpretation.

- Inter-cause Independence : Variables A and B are inter-causally independent iff the subgraph, G^+_A, formed by all the variables causally influencing A is disjoint to the subgraph, G^+_B, formed by all the variables causally influencing B.

- Inter-symptom Independence : Variables A and B are inter-symptomically independent iff the subgraph, G^-_A, formed by all the variables diagnostically influencing A is disjoint to the subgraph, G^+_b, formed by all the variables diagnostically influencing B.

- Total Independence : Variables A and B are (totally) independent iff A and B are inter-causally independent and inter-symptomically independent, i.e, the subgraph containing variable A is disjoint to the subgraph containing variable B.

The distinction of inter-cause from cross-generation and inter-symptom independence is one of the unique features of CONVINCE's causal network. The distinction is based not on arbitrary assumptions but upon the conformity to common modes of reasoning. This sets our interpretation of structural relationships apart from other schemes wherein assumptions of independence are introduced solely to obtain a tractable solution. The brute force independence or conditional independence assumptions, even in the scheme obtained by applying the least information principle [Lewis 59, Brown 59, Duda 79, Dalkey 81], are invariant to the direction of causality and hence, do not take into account this aspect of human reasoning.

Chapter 3

INFERENCE PROCEDURE

3.1 OVERVIEW

The integration of new pieces of information into the existing body of knowledge constitutes a fundamental problem in a number of decision-making tasks such as situation assessment, diagnosis, pattern recognition and speech understanding. Knowledge-based expert systems and decision support systems must handle this problem both to achieve an expert level of performance and to derive valid recommendations. This chapter addresses the issues of propagating the impact of new evidence and beliefs efficiently through a hierarchically organized inference network. The inference procedure described here models both causal and diagnostic modes of reasoning[1] simultaneously, and is a generalization of the Bayesian methods previously applied to trees[DDI 73, Pearl 82] toward developing a class of hierarchical networks suitable for the modeling of

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1. See Section 1.1 for definitions.

multiple causes.

The tree representation insists that only one variable be considered a cause of any other variable. This restriction simplifies computation and avoids the problem of maintaining consistency among interrelated variables. However, its representational power is so restricted that many real problems cannot be modeled naturally. In order to comply with the requirements imposed by the tree structure, we must group together all the causal factors as the set of states of one single variable. By contrast, when people associate a given observation with multiple potential causes, they weigh one causal factor against another as independent variables, each pointing to a specialized area of knowledge. As an illustration, consider again the following situation (see Chapter 2).

> Mr. Holmes received a telephone call from his neighbor notifying him that she heard a burglar alarm sound from the direction of his home. As he was preparing to rush home, Mr. Holmes recalled that last time the alarm had been triggered by an earthquake. On his way driving home, he heard a radio newscast reporting an earthquake 200 miles away.

Mr. Holmes perceives two episodes which may be potential causes for the alarm sound, an attempted burglary and an earthquake. Even though these two events are a priori independent and so, not mutually exclusive, still the radio anouncement reduces the likelihood of a burglary by "explaining away" the alarm sound. Moreover, the two causal events are perceived as individual variables each pointing to a separate frame of knowledge. The tree representation, on the other hand, would force us to cluster the two causal events into a single four-state variable called CAUSE, which includes all the possible combinations of the events BURGLARY, EARTHQUAKE and their negations.

The computational scheme described here uses Bayes' calculus to model that kind of interaction among causes in addition to the ususal interaction among diagnostic indicators. Belief parameters are identified and an efficient updating scheme is developed by exploiting inter-cause and inter-symptom independences. the updating scheme modifies all the beliefs upon receiving new evidence in a single pass through the network, avoiding the infinite relaxations.

3.2 OTHER APPROACHES TO THE INFERENCE PROBLEM

The derivation of inference with uncertainty amounts to the calculation of the posterior probability distribution of a target variable after observing relevant pieces of evidence. Formally, an inference problem is a search for the posterior probability distribution $\underline{P}(T|E)$, where T stands for a target variable and E stands for the observed evidence. Usually, E consists of more than one piece of evidence, namely E_1, E_2, ..., E_n.

Direct elicitation of high order probability distributions is practically impossible because the required data grows exponentially with their order. Even if one endured the time and storage required for assessing a high order distribution, probably it would be unreliable because complex events, involving many variables, are not apprehendable by ordinary people with their "bounded rationality"[Simon 57].

However, since the local relations through intervening variables are commonly assessible with a greater confidence than direct relations between the target hypotheses and evidence set, various inference procedures have been devised to derive the posterior probability distribution from a set of known probabilities involving intermediate variables. The set of known probabilities in most practical applications consists of low order probabilites, involving a small number of variables in their description, even though the posterior probability distribution is of a high order.

Bayes' theory has been the most widely accepted method for the last two centuries for combining pieces of evidence in order to update the posterior probability distribution. Formally, a brute force application of the Bayes' rule would require a large amount of data, such as the joint probability distribution for all of the events considered, so that, a number of assumptions are introduced for practical purposes to restrict the interactions among variables. These include assumptions of independence, conditional independence, mutual exclusiveness, and exhausitivity. An approach like the Least Information Principle makes these assumptions implicitly.

A typical example of a Bayesian approach is found in DDI's hierarchical tree method[DDI 73, Pearl 82]. In a hierarchical tree, nodes represent variables and links represents correlations between a pair of variables in the form of the conditional probability of an evidence given a hypothesis. It is assumed that any two variables are conditionally independent given an intermediate variable between them. Bayes' rule, together with this assumption, can be used to calculate the distributions of all variables in the tree by using the prior probability distribution on the root node and the conditional probabilities relating variables in the tree, Although computation on the hierarchical tree structure is simple and the problem of maintaining consistency is avoided, its representational power is very restricted.

PROSPECTOR is a computer program for mineral exploration[Duda 79]. This system accomodates the uncertainty that is often associated with geological observations and conclusions by assigning a probability to every assertion in its model. Bayesian calculus is used to guide the updating of these probabilities as evidence is acquired. That is, the builder of a model assigns both a prior probability value to every event and some correlations between events in the form of a premise-consequent rule with its associated conditional probability. The system updates these probabilities from their prior to their posterior values as the user provides more information. To cope with the requirements of Bayesian updating, Duda also introduced the assumption that pieces of evidence are conditionally independent under a given hypothesis. In cases where the interactions between variables are so complex that they violate the the conditional independence, some logical connectives such as conjuction, disjunction and negation are used. Computation of the probabilities for the logical connectives is performed according to the conventions of fuzzy set theory. Since the model builder specifies the prior $P(H)$ of each hypothesis H, the prior $P(E)$ of the evidence E, and their correlation in the form of conditional probability matrix $P(H|E)$, inconsistency may occur when the values are elicited independently. To remedy the inconsistency problem, a piecewise linear interpolation method is used with the fixed points being $P(H|E)$, $P(H)$ and $P(E)$. Furthermore, two or more uncertain pieces of evidence are combined heuristically into a hypothesis according to a formula called L' heuristic in that the posterior odd $O(H|E)$, after observing evidence E consisting of E_1, E_2, ..., E_n, is computed by :

$$(3\text{-}1) \quad O(H|E) = [\prod_i L_i]\, O(H)$$

where

$$(3\text{-}2) \quad L_i = O(H/E) \,/\, O(H).$$

To sum to this point, PROSPECTOR's approach is based on Bayesian probability theory with a conditional independence assumption, but some ad hoc approaches were blended in order to extend its modeling power. However, a later report[Duda 79] included a review of its inference scheme from a pure Bayesian viewpoint and suggested the use of the Least Information Principle. Since a brute force implementation of the Least Information Principle is computationally infeasible even with a moderate number of variables, an approximation scheme has been proposed.

The Least Information Principle has been developed to estimate a high order probability distribution using the partial information contained in the lower order component distributions[Lewis 59]. Since a conditional probability can always be derived from the joint probabilties of the same order, such that $P(A|B)$ is derivable from $P(A,B)$, the estimate of high-order joint probability distribution can be used for the solution of the inference problem. When we have no information about the events that a probability distribution admits, it is quite reasonable to assume that the probability distribution is flat so that each event is equally likely to happen. The Least Information Principle supports this intuition. An information measure, $I(P)$, of a probability distribution P, is defined as:

$$(3\text{-}3) \qquad I(P) = \log N - H(P)$$

where N is the number of events of the distribution and $H(P)$ is the entropy of the distribution:

$$(3\text{-}4) \qquad H(P) = - \sum_i P_i \log P_i .$$

The information measure has the property that:

$$(3\text{-}5) \qquad \emptyset \leq I(P) \leq \log N.$$

$I(P)$ is zero if P is the uniform distribution while $I(P)$ becomes maximum, that is, $\log N$, when the distribution is peaked at a point i with $P = 1$. The Least Information Principle is equivalent to asserting that "an unknown probability distribution should be approximated by the one that contains the minimal information among the candidate distributions satisfying all known constraints" or selects "the maximum entropy distribution among the ones compatible with what is known about". The selection of the least information distribution amounts to optimizating a non-linear objective function subject to linear constraints. Since this non-linear programming problem can be solved only by iterative procedures such as the convex simplex method, the time required for this computation exceeds our allowance in most nontrivial cases.

Recently a number of heuristic approaches have been developed for solving the inference problem, mostly in the artificial intelligence field. These heuristic approaches appeal to our intuitions and sometimes offer the only practical solution where there is a lack of sufficient statistical data. However, although these procedures are reported to "work well" in particular domains, their performance in others is not predictable because the assumptions underlying their "success" have not been explicated. Therefore, it is always necessary to check the validity of these heuristic inference procedures in each specific domain through an extensive testing.

MYCIN is a medical consultation system which was developed to advise physicians and medical students in the treatment of infectious diseases[Shortliffe 76]. MYCIN's knowledge is expressed in a set of rules which are in the form of premise-consequence. MYCIN associates with each rule a certainty factor CF which takes on a value between Ø and 1, and represents the added degree of belief for the consequence. Every hypothesis is associated with a measure of belief MB and a measure of disbelief MD, each ranging from Ø to 1, summarizing all the positive and negative evidences, respectively. The MB and MD are maintained separately, insuring that the rule "A implies B with probability X" should not be inverted in the traditional probabilistic sense to yield "A implies NOT B with probability (1-X)". When a premise of a rule is uncertain, MB and MD are reduced according to the degree of the uncertainty. When a premise is in a conjunctive or disjunctive form, the measures of MB and MD are calculated by the min or max operation on their components following the fuzzy set tradition. The combination of two premises E_1, E_2 leading to one common consequence H is given by the following formulas:

$$MB(H|E_1,E_2) = \begin{cases} \emptyset, & \text{if } MD(H|E_1,E_2) = 1 \\ MB(H|E_1) + MB(H|E_2)[1-MB(H|E_1)], & \text{otherwise.} \end{cases}$$

Similarly,

$$MD(H|E_1,E_2) = \begin{cases} \emptyset, & \text{if } MB(H|E_1,E_2) = 1 \\ MD(H|E_1) + MD(H|E_2)[1-MD(H|E_1)], & \text{otherwise.} \end{cases}$$

Finally, a CF for the hypothesis is obtained by combining the MB and MD as:

$$CF(H|E_1,E_2) = MB(H|E_1,E_2) - MD(H|E_1,E_2)$$

with the CF of a hypothesis taking a value between -1 and 1. MYCIN represents the relations between two variables only in a diagnostic form where P(Hypothesis|Evidence), regardless of their causality relations. As we discussed in the previous chapter, the validity of assessing these quantifiers can be improved by exploiting causality relations.

INTERNIST is a computerized diagnostic program which emphasizes a very broad coverage of clinical diagnostic situations[Pople 75]. The INTERNIST data base associates with every possible disorder D_i a set of manifestations $\{M_j\}$. For every M_j listed under D_i, two likelihoods are entered: $L(D_i|M_j)$ and $F(M_j|D_i)$. $L(D_i|M_j)$, the evoking strength, is the likelihood that if manifestation M_j is seen, its cause is D_i. It is assessed on a scale of 0 to 5. $F(M_j|D_i)$, the frequency, is the likelihood that a patient with a confirmed diagnosis D_i would exhibit M_j. Note that $F(M_j|D_i)$ is quite analogous to the conditional probability $P(M_j|D_i)$, while the evoking strength is like a posterior probability $P(D_i|M_j)$. However, INTERNIST manipulates these numbers in an ad hoc fashion without any theoretical guidelines. A score which represents its degree of confirmation is computed for each hypothesis by summing up the evoking strengths of all its evidence, and by substracting the sum of the frequencies of those manifestations which are known to be absent, and the weight of importance of each significant finding which is not explained by either the diagnosis or some other confirmed diagnosis. Thus, the evocative finding and confirmed consequences of a diagnosis count in its favor, while expected findings which are known to be absent and reported findings which are unexplained count against it.

3.3 PEARL'S WORK IN INFERENCE TREES

Pearl has developed an elegant belief propagation scheme which is applicable to tree-structured hierarchical networks[Pearl 82b]. This scheme will be briefly reviewed here since the propagation scheme described in the next section resembles and generalizes it in many aspects. Pearl has restricted his attention to a special kind of causality tree where only one causal predecessor is allowed for any given manifestation variable. He interprets the existence of a single path between any pair of nodes in the traditional Markovian sense such that if X is not a descendant of Y, then the influence of X on node Y is completely summarized by X's influence on the father of Y. This encompasses both the traditionally used "conditional independence" assumption in characterizing relationships among siblings as well as the cross-generation independence between grandparents and grandchildren. Accordingly, he derives the formula for combining influences from above and below a given node A as:

$$(3\text{-}5)\qquad P(A|D^{+}_{A},D^{-}_{A}) = \beta\, P(D^{-}_{A}|A)\; P(A|D^{+}_{A}).$$

where D^{+}_{A} represents data above node A and D^{-}_{A} represents data below A.

Moreover, assuming that the vectors:

$$(3\text{-}6)\qquad \underline{\lambda}(A) = P(D^{-}_{A}|A)$$

and

$$(3\text{-}7)\qquad q(A) = P(A|D^{+}_{A})$$

are stored with each node of the tree, the influence of new information can spread through the tree using local communication between variables. The propagation can be summarized as follows.

1. Each node computes two message vectors: $\underline{P}$ and $\underline{r}$. $\underline{P}$ is sent to every son while $\underline{r}$ is delivered to the father. The message $\underline{P}$ represents the current probability distribution of the sender and is computed by:

$$(3\text{-}8) \quad P(A_i) = \beta \cdot \lambda(A_i)\ q(A_i)$$

and $\underline{r}$ is computed from $\underline{\lambda}$ using matrix multiplication:

$$(3\text{-}9) \quad \underline{r} = \underline{M}\ \underline{\lambda}$$

where $\underline{M}$ is the matrix quantifying the link to the father.

2. When node A is called to update its parameters, it inspects simultaneously the message $\underline{P}(B)$ from its father B and the messages $\underline{r}_1$, $\underline{r}_2$, ..., from its sons. It then updates $\underline{\lambda}$ and $\underline{q}$ as:

$$(3\text{-}10) \quad \lambda(A_i) = \prod_k (\underline{r}_k)_i$$

and

$$(3\text{-}11) \quad q(A_i) = \beta \sum P(A_i|B_j)/(\underline{r}')_j$$

where β is a normalization constant and $\underline{r}'$ is the last message sent from A to B.

This updating scheme is shown in the Figure 3-1 where the multiplications and division of two vectors stand for the term-by-term operations.

The inference scheme which will be described in the next section generalizes Pearl's scheme by both permitting more than one causal factor for a given manifestation to be

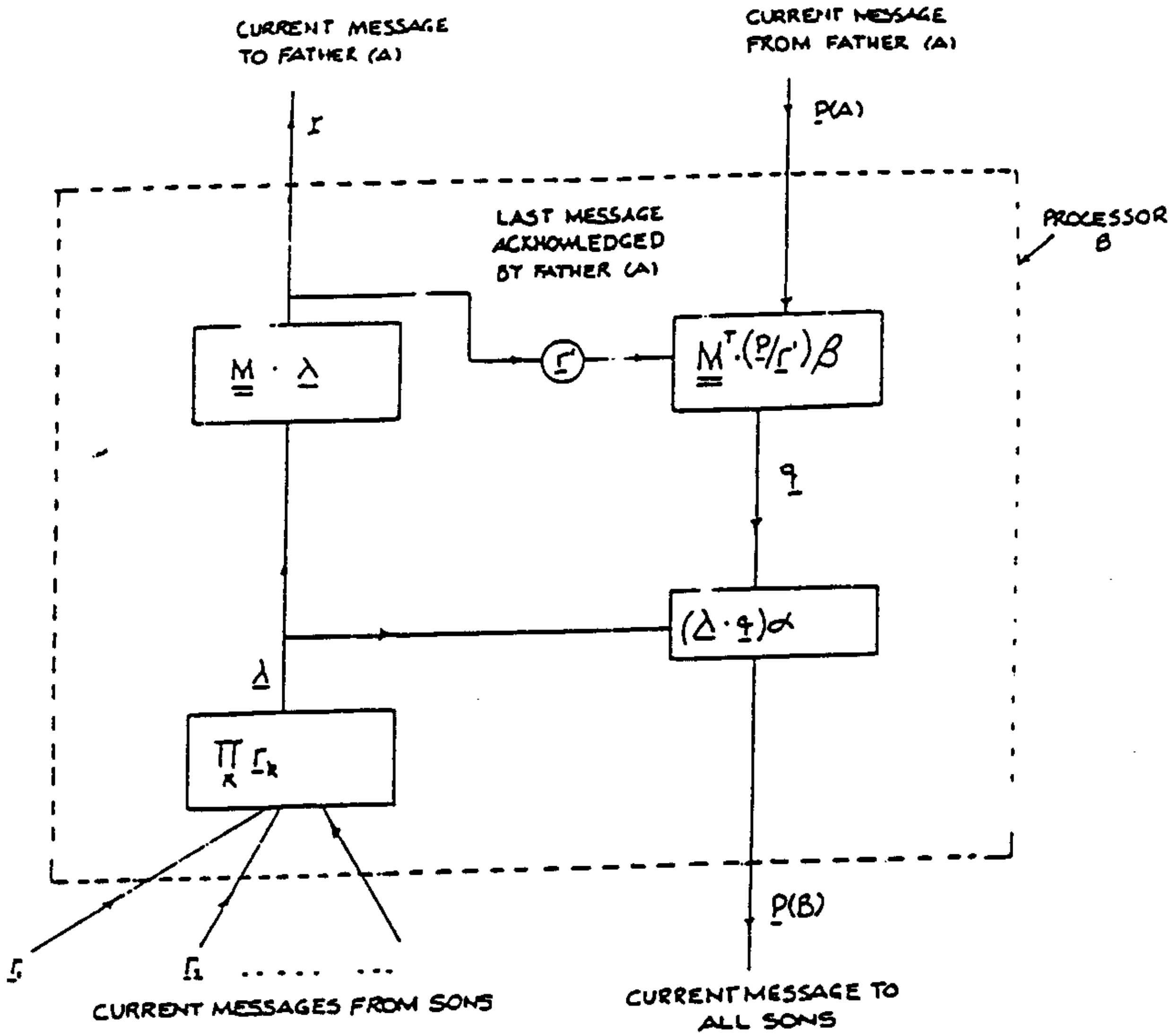

Figure 3-1 : Pearl's Belief Parameter Updating Scheme.

specified and representing these as individual nodes. Interactions among causes are modelled more naturally by this extension. Furthermore, the scheme described in the next section uses the belief parameters stored at each node both in computing the belief distribution of that node and as communicating messages to neighbors.

3.4 BELIEF PARAMETERS

Consider the network of Figure 3-2. The strength of belief $BEL(A_i)$ on A_i should, at any given time, reflect the entire data observed so far, i.e., data from subgraphs G^+_{BA}, G^+_{CA}, G^-_{AX} and G^-_{AY}. Hence, we write:

$$(3\text{-}12) \quad BEL(A_i) = P(A_i | D^+_{BA}, D^+_{CA}, D^-_{AX}, D^-_{AY}).$$

According to Bayes' rule and the cross-generation assumption (2-2):

$$(3\text{-}13) \quad BEL(A_i) = \alpha\, P(A_i | D^+_{BA}, D^+_{CA})\, P(D^-_{AX}, D^-_{AY} | A_i)$$

where α is a normalization constant[2]. Further, applying inter-cause and inter-symptom independence, Eqs. (2-6) and (2-7), yields:

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2. We assume that α is chosen to make $\sum BEL(A_i) = 1$. However, one may relax this constraint to represent the degree of ignorance, as in Dempster-Shafer system [Shafer 76, Barnett 81, Garvey 81].

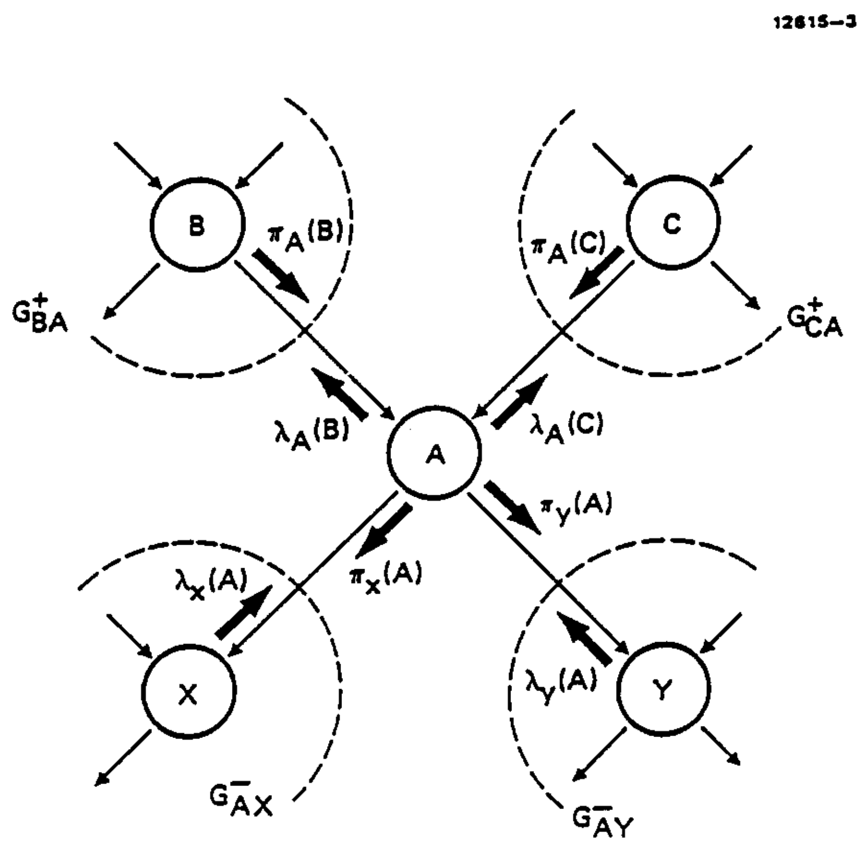

Figure 3-2 : A Fragment of a Causal Network

$$(3\text{-}14)\quad BEL(A_i) = \alpha\, P(D^-{}_{AX}\,|A_i)P(D^-{}_{AY}\,|A_i) \cdot \sum_{jk} P(A_i|B_jC_k)P(B_j|D^+{}_{BA})P(C_k|D^+{}_{CA}).$$

Equation (3-14) suggests that the probability distribution of each variable A in the network could be computed if three parameters are made available:

1. the current strength of the causal evidence, PI, contributed by each incoming link to A, where:

$$(3\text{-}15)\quad \mathbf{PI}_A(B_j) = P(B_j|D^+{}_{B,A})$$

2. the current strength of the diagnostic evidence, **LAMBDA**, contributed by each outgoing link from A, where:

$$(3\text{-}16)\quad \mathbf{LAMBDA}_X(A_i) = P(D^-{}_{A,X}|A_i)$$

3. the fixed conditional probability tensor[3], P(A|B,C), which relates the variable A all combinations of to its immediate causes B and C.

Accordingly, in the propagation scheme which we have devised, we let each link carry two dynamic parameters, PI and **LAMBDA**, and let each node store the information

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3. A tensor is an extension and generalization of a vector and matrix into higher orders. A vector is a tensor of order one, a matrix is a tensor of order two.

contained in $P(A|B,C)$.

We will use **LAMBDA** without subscripts to refer to the multiplicative integration of all the **LAMBDAs** pertaining to a given node, such that:

$$(3\text{-}17)\quad \mathbf{LAMBDA}(A_i) = \mathbf{LAMBDA}_X(A_i)\ \mathbf{LAMBDA}_Y(A_i).$$

Similarly, PI without subscripts will be used to refer to the combined influence from all the predecessors of a node or:

$$(3\text{-}18)\quad \mathbf{PI}(A_i) = \sum_{jk} P(A_i|B_j,C_k)\ \mathbf{PI}_A(B_j)\ \mathbf{PI}_A(C_k).$$

Thus, we can write:

$$(3\text{-}19)\quad \mathbf{BEL}(A) = \alpha\,\mathbf{LAMBDA}(A)\ \mathbf{PI}(A)$$

We will omit the normalizing constant α for simplicity. The constant can be recalculated whenever needed. PI(A) represents the anticipatory support attributed to A by all its predecessors, its causally influencing data; **LAMBDA**(A) represents the evidential support received by A from all its descendants, its diagnostically influencing data. As Pearl pointed out [82b], the equation (3-19) is a generalization of the Bayesian odd-likelihood multiplication rule:

$$(3\text{-}20)\quad O(H|E) = \lambda(E)\ O(H)$$

with $\lambda(E) = P(E|H)/P(E|\text{ NOT }H)$ known as the likelihood ratio, and $O(H) = P(H)/P(NOT\ H)$ known as the prior odd. Equation (3-19) also explicates the meaning of the prior probability term $P(H)$, which represents likelihood of a variable state, given all its causally influencing data.

3.5 PROPAGATION OF INFORMATION THROUGH THE NETWORK

Our task is now to prescribe how the influence of new information spreads through the network assuming that the vectors **LAMBDA** and **PI** are stored with each link and the conditional probability tensor is stored with each node. Let us recall some multiplication operators defined on tensors before we present the updating formulae.

1. The outer-product of a tensor A of order M and a tensor B of order N becomes a tensor of order M+N in which an element of tensor C is the product of the corresponding elements of A and B. We will use the symbol '$\otimes$' as the outer-product operator.

2. The inner-product of tensors A and B is defined to be the tensor formed from the outer product of tensors A and B by properly summing over the indices that appear both in A and B. We will use the symbol '$\bullet$' as the inner product operator. For example, the inner product of a tensor T_{12345} of order five and a tensor

R_{34} of order two becomes a tensor S_{125} of order three.

3. The <u>term-by-term product</u> is defined only between two tensors of the same order and dimension. Each element of the product tensor is the product of the corresponding elements of the two tensors. We will use the symbol 'o' as the term-by-term product operator.

Updating LAMBDA

If we assume that B and C form a super variable which admits all combinations of the states of B and C, then:

$$
\begin{aligned}
\text{(3-21)}\quad \mathrm{BEL}(B_i) &= \sum_j \mathrm{BEL}(B_i C_j) \\
&= \sum_j P(B_i C_j \mid D^+_{BA}, D^+_{CA})\ P(D^-_{AX}, D^-_{AY} \mid B_i C_j) \\
&= \sum_j [\ P(B_i \mid D^+_{BA}) P(C_j \mid D^+_{CA}) \\
&\qquad \sum_k P(D^-_{AX} \mid A_k) P(D^-_{AY} \mid A_k) P(A_k \mid B_i C_j)] \\
&= \sum_j [\ PI_A(B_i) PI_A(C_j) \\
&\qquad \mathrm{LAMBDA}_X(A_k) \mathrm{LAMBDA}_Y(A_k) P(A_k \mid B_i C_j)]
\end{aligned}
$$

and, at the same time:

$$\text{(3-22)}\quad \mathrm{BEL}(B_i) = PI_A(B_i)\mathrm{LAMBDA}_A(B_i).$$

Equating Eq (3-21) and (3-22) yields:

$$(3\text{-}23)\quad \mathbf{LAMBDA}_A(B_i) = \sum_j [PI_A(C_j) \sum_k \mathbf{LAMBDA}_X(A_k)\mathbf{LAMBDA}_Y(A_k)P(A_k|B_iC_j)].$$

Euation (3-23) can be rewritten by using vector notations and the product operators as:

$$(3\text{-}24)\quad \mathbf{LAMBDA}_A(B) = PI_A(C)\cdot((\mathbf{LAMBDA}_X(A)\circ \mathbf{LAMBDA}_Y(A))\cdot P(A|BC))$$

Equation (3-24) shows that only three parameters, in addition to the conditional probability tensor $\underline{P}(A|B,C)$, need to be involved in updating the diagnostic parameter vector $\underline{\mathbf{LAMBDA}}_A(B)$ from A to B: $\underline{\mathbf{PI}}_A(C)$, $\underline{\mathbf{LAMBDA}}_X(A)$ and $\underline{\mathbf{LAMBDA}}_Y(A)$. This is expected since $\underline{\mathbf{PI}}_A(B)$ stands for $\underline{P}(B|D^-_{BA})$ and D^-_{BA} is completely summarized by the above three parameters (see Figure 3-2).

Updating PI

The rule for updating the causal parameter $PI_X(A)$ can be obtained from the equation:

$$\begin{aligned}(3\text{-}25)\quad PI_X(A_i) &= P(A_i|D^+_{BA},D^+_{CA},D^-_{AY})\\ &= P(D^-_{AY}|A_i)\\ &\quad [\sum_{jk} P(A_i|B_jC_k)P(B_j|D^+_{BA})P(C_k|D^+_{CA})]\\ &= \mathbf{LAMBDA}_Y(A_i)\\ &\quad [\sum_{jk} P(A_i|B_jC_k)\ PI_A(B_j)PI_A(C_k)].\end{aligned}$$

By using vector notations and the product operators, the equation is written as:

$$(3\text{-}26)\quad \mathbf{PI}_X(A) = \mathbf{LAMBDA}_Y(A) \circ \left[P(A|BC) \cdot (\mathbf{PI}_A(B) \otimes \mathbf{PI}_A(C)) \right].$$

Thus, similar to $\underline{\mathbf{LAMBDA}}_A(B)$, $\underline{\mathbf{PI}}_X(A)$ is determined also by three neighboring parameters: $\underline{\mathbf{LAMBDA}}_Y(A)$, $\underline{\mathbf{PI}}_A(B)$ and $\underline{\mathbf{PI}}_A(C)$.

Equations (3-24) and (3-26) also demonstrate that a perturbation of the causal parameter, **PI**, will not effect the diagnostic parameter, **LAMBDA**, on the same link, and vice versa. Therefore, any perturbation of beliefs due to new evidence propagates through the network and is absorbed at the boundary without reflection. A new equilibrium state will be reached after a finite number of updates which, in the worst case, is equal to the diameter of the network.

Equation (3-24) reveals that if no data is observed below A where all **LAMBDAs** to A are unit vectors, then all **LAMBDAs** from A are also unit vectors. This means that evidence gathered at a node does not influence its "spouses" until their common "son" gathers diagnostic evidence. In Mr. Holmes' case, for example, seismic data pertaining to earthquakes would not have influenced the likelihood of burglary prior to receiving the neighbor's telephone call. It is a pleasing characteristic. Otherwise, a node may gather support through purely mental constucts void of diagnostic support.

A node which has no predecessor needs a special parameter unless it is a data node. Since no causal influence is available from its predecessors, it requires an external parameter to summarize the background, a priori[4] knowledge pertaining to that node, thus serving the classical role of subjective prior probability.

Generalization of Equations (3-24) and (3-26) for more than two causal factors and more than two sets of manifestations is straightforward. The bottom-up propagation parameter, **LAMBDA**, can be defined as:

$$(3\text{-}27)\quad \mathrm{LAMBDA}_A(B^i) = \sum_{p} \prod_{k \neq i} \mathrm{PI}_A(B^k) \left[\sum_{\substack{B^1,\ldots,B^n \\ \neq B^i}} P(A_p \mid B^1 \ldots B^n) \prod_{l} \mathrm{LAMBDA}_X{}^l(A_p) \right]$$

and the top-down propagation parameter, PI, as:

$$(3\text{-}28)\quad \mathrm{PI}_X{}^j(A) = \left[\sum_{\substack{B^1,\ldots,B^n \\ \neq B^j}} P(A \mid B^1 \ldots B^n) \prod_{l} \mathrm{PI}(B^l) \right] \left[\prod_{k \neq j} \mathrm{LAMBDA}_X{}^k(A) \right].$$

----

4. According to Webster's New World Dictionary, "a priori" means "from cause to effect".

This updating scheme is shown schematically in the diagram in Figure 3-3. The left half represents the process of upward propagation and the right half represents that of downward propagation. The combined influence on node A from its M predecessors, denoted by $\underline{\pi}_A$ in the figure, is computed by the inner-product of the conditional probability tensor (denoted by $P_{A123...M}$ and is stored at the node) with the outer-product of the causal parameters (denoted by $\underline{\pi}_j$) from its predecessors. The combined influence on node A from its N successors, denoted by $\underline{\lambda}_A$, is computed by the term-by-term product of the diagnostic parameters (denoted by $\underline{\lambda}_j$) from its successors. The belief distribution of the node, BEL(A), is computed by taking the term-by-term product of $\underline{\pi}_A$ and $\underline{\lambda}_A$. The new downward propagation message for each successor j, denoted by $\underline{\pi}'_j$, is computed by taking the term-by-term product of the combined causal influence $\underline{\pi}_A$ and combined diagnostic influences from other than the successor j. This computation is performed elegantly by nullifying the influence from that successor, i.e., by the term-by-term division of the total belief BEL(A) by the upward message that has been sent from that successor.

The new upward propagation message to each predecessor i, denoted by $\underline{\lambda}'_i$, is computed by taking the inner product of $\underline{\lambda}_A$ with $P_{A123...M}$, and then with the outer-product of all the downward mesages other than i (denoted by $_{\neg i}\pi_{123...M}$). The

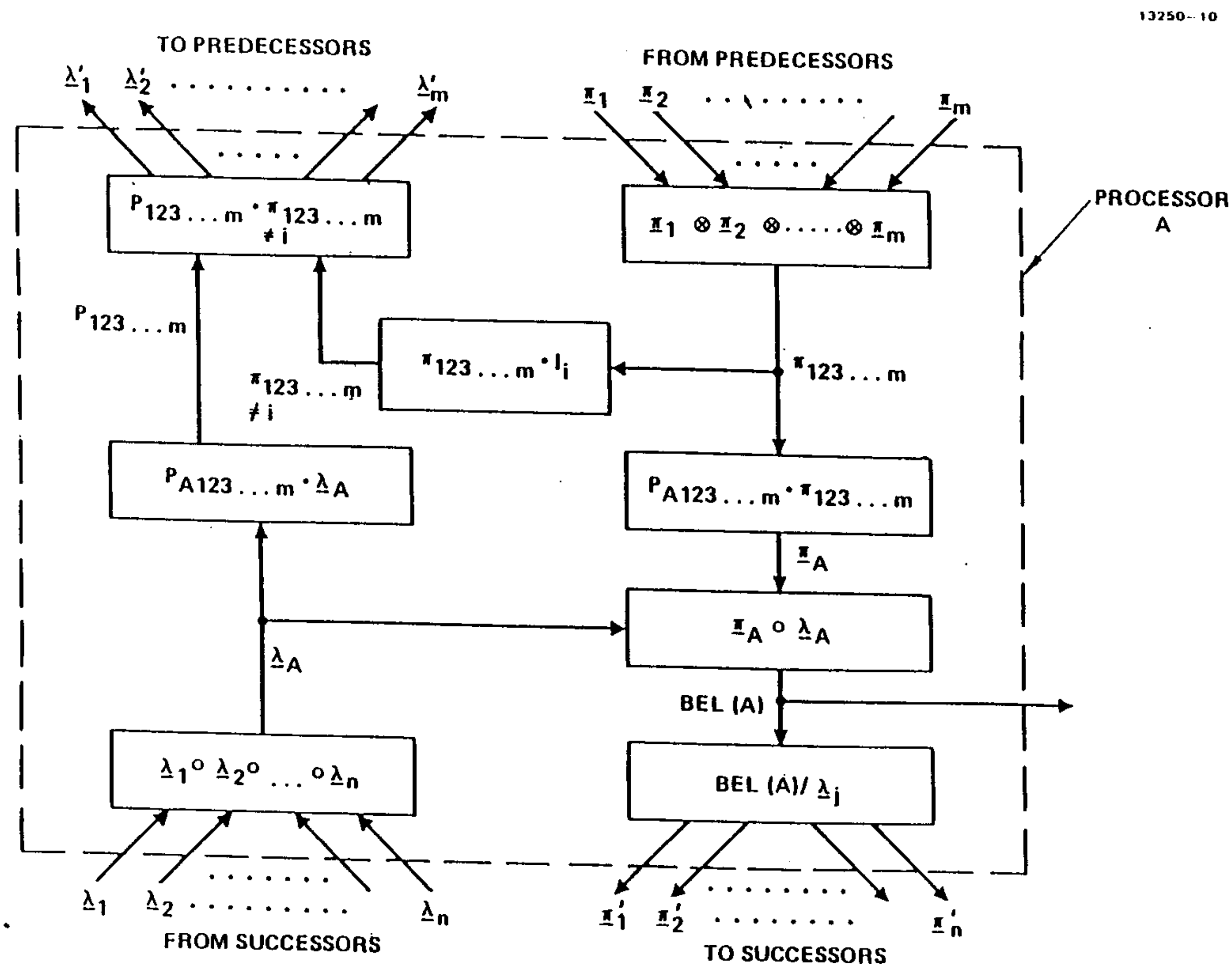


Figure 3-3 : Belief Parameter Updating Scheme term $\pi_{*i\,123...M}$ is also computed elegantly by nullifying the i-th predecessor's influence from the total outer-product $\pi_{123...M}$. In this case, the nullification is achieved by the inner-production of the unit vector by setting it to i-th index.

We have introduced a formalization for the interaction among multiple causes which reflects the way people often view causal relationships. Based on this formulation, we have extended a tree representation to a class of hierarchical networks capable of modeling multiple causes while still maintaining the computational efficiency provided by the tree representation. This formulation shows that belief parameters can be updated in a single pass by local computations and in strict conformity to probability theory.

3.6 APPROXIMATING THE CONDITIONAL PROBABILITY TENSOR

We have shown that the computation of beliefs in a network with multiple-parent variables needs high order conditional probabilities. In principle, the specification of $\underline{P}(A|B,C)$ requires a table with one entry for each state combination of the variables A, B and C. Needless to say, such a table is rather troublesome to obtain from experts due to its size. For this reason, it is necessary to approximate the high-order conditional probability $\underline{P}(A|B,C)$ from the <u>pairwise</u> relations $\underline{P}(A|B)$ and $\underline{P}(A|C)$.

A description of a state at a given level of detail is an aggregation of states of the next more detailed level[Patil 81]. For example, the state 'alarm' in Mr. Holmes' case is a summarization of its more detailed level states, 'alarm caused by burglar' and 'alarm caused by earthquake'. Moreover, either a burglar or an earthquake may cause the alarm sound separately, while the state 'alarm sound' is false when both 'alarm sound cause by a burglar' and 'alarm sound caused by an earthquake' are false. In such a case, we say that the state 'alarm sound' <u>dominates</u> its complement state. Note that the dominance relationship is a characteristic of a variable itself, not of the strength of causal relations with its neighbors.

Imagine an alarm which has two rings in it. One of them, called ring1, is designed to detect burglaries and the other, ring2, is designed to detect earthquakes. The ring1 detects burglaries with probability $P(A1|\ B)$ and falsely alarms burglaries with probability $P(A1|\ \text{not } B)$. Similarly, the ring2 detects earthquakes with probability $P(A2|\ E)$ and falsely alarms earthquakes with probability $P(A2|\ \text{not } E)$. Since the state ALARM SOUND dominates its complement state, we can write:

$$\text{(3-29)} \quad P(\text{ALARM SOUND} \mid B, E) = P(A1, A2 \mid B,E) + P(A1, \text{not } A2 \mid B,E) + P(\text{not } A1, A2 \mid B,E) \quad ,$$

i.e., sounding of any of the rings constitutes the state ALARM SOUND. Since the causes of alarming at ring1 and ring2 are independent, we write:

$$\text{(3-30)} \quad P(A1,A2 \mid B,E) = P(A1 \mid B)\, P(A2 \mid E)$$

$$P(A1, \text{not } A2 \mid B,E) = P(A1 \mid B)\, P(\text{not } A2 \mid E)$$

$$P(\text{not } A1, A2 \mid B,E) = P(\text{not } A1 \mid B)\, P(A2 \mid E)$$

$$P(\text{not } A1, \text{not } A2 \mid B,E) = P(\text{not } A1 \mid B)\, P(\text{not } A2 \mid E).$$

Assuming that $P(A1 \mid B)$ is equal to $P(\text{ALARM SOUND} \mid B)$ and $P(A2 \mid B)$ is equal to $P(\text{ALARM SOUND} \mid B)$, the following formula is derived.

$$\text{(3-31)} \quad P(\text{ALARM SOUND} \mid B,E)$$
$$= P(\text{ALARM SOUND} \mid B)\, P(\text{ALARM SOUND} \mid E)$$
$$+ P(\text{ALARM SOUND} \mid B)\, P(\text{not ALARM SOUND} \mid E)$$
$$+ P(\text{not ALARM SOUND} \mid B)\, P(\text{ALARM SOUND} \mid E)$$

and

$$P(\text{not ALARM SOUND} \mid B,E)$$
$$= P(\text{not ALARM SOUND} \mid B)\, P(\text{not ALARM SOUND} \mid E) \ .$$

In words, the strength of the belief of an aggregated state is computed by the sum of the beliefs committed to its component states. This computation is illustrated in Figure 3-3 for a binary variable case and in Figure 3-4 for a trinary variable case. In Figure 3-4, beliefs supported by two causal states B_k and C_l are combined. The vertical axis represents the belief distribution of A supported by B_k, and the horizontal axis represents that of A supported by C_l. If we assume that A_i dominates A_j for $i < j$, then the combining formula is:

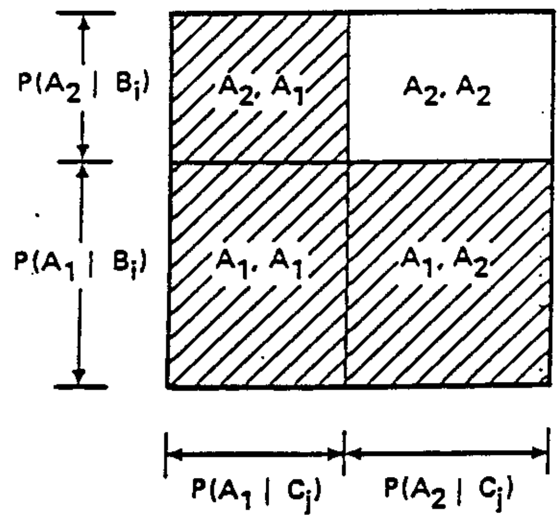
$$(3\text{-}32) \quad P(A_i|B_k,C_l) = \alpha \sum_{q>i} P(A_i|B_k)P(A_q|C_l) + \sum_{q>i} P(A_q|B_k)P(A_i|C_l)]$$

where is a normalization constant. Equation (3-32) means that the regions of conflicting labels are resolved by the dominance relation.

The dominance relation may not hold for some variables. For those, the regions of conflicting labels are ignored and the ratio of the diagonal regions serves to produce belief distribution (see Figure 3-5 for a binary variable case and Figure 3-6 for a trinary variable case). For trinary variable case,, the combining formula becomes:

$$(3\text{-}33) \quad P(A_i|B_k,C_l) = \alpha P(A_i|B_k)P(A_i|C_l).$$

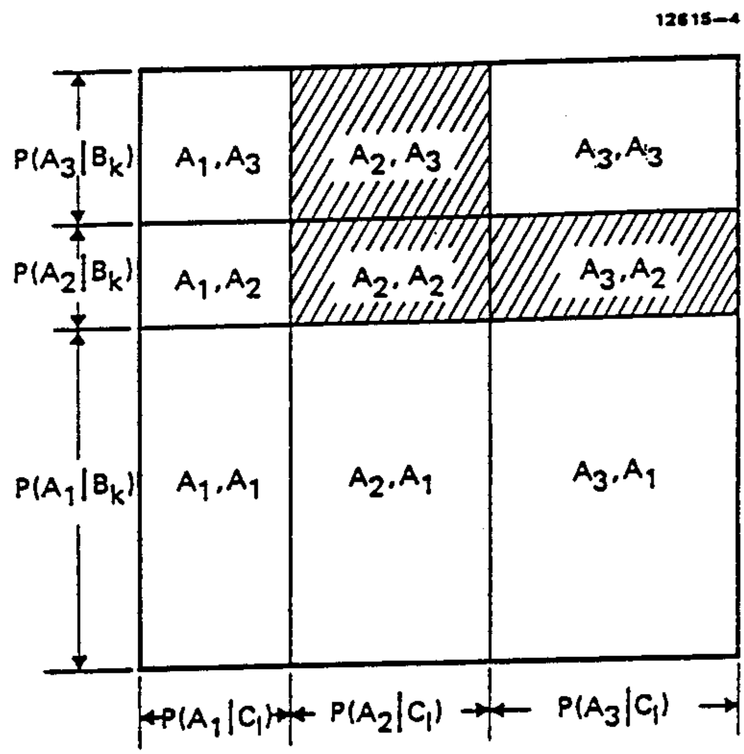
Note that this formula is same as Dempster's rule of combination known as "orthogonal sum", except in the treatment of ignorance[Shafer 76, Garvey 81, Barnett 81].



$$
\begin{aligned}
P(A_1 \mid B_i, C_j) = \; & P(A_2 \mid B_i)\, P(A_1 \mid C_j) \\
& + P(A_1 \mid B_i)\, P(A_2 \mid C_j) \\
& + P(A_1 \mid B_i)\, P(A_2 \mid C_j) \\
P(A_2 \mid B_i, C_j) = \; & P(A_2 \mid B_i)\, P(A_2 \mid C_j)
\end{aligned}
$$

Figure 3-4 :

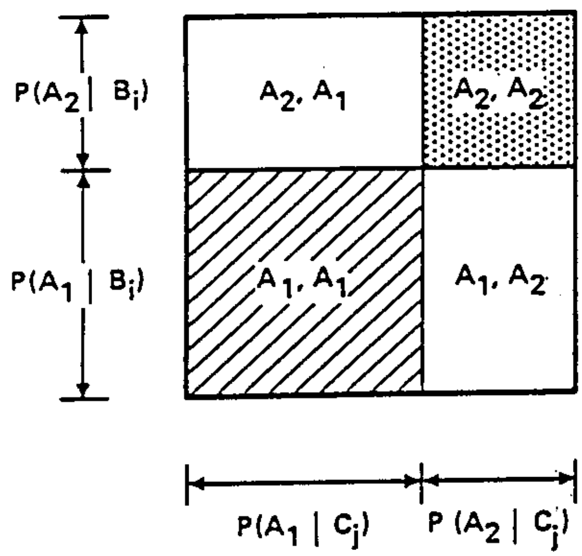
Approximation of Conditional Tensor - Binary variable when dominance relation holds.



- A_i DOMINATES A_j FOR $i < j$.
- SHADED AREA CORRESPONDS TO THE BELIEF OF A_2 GIVEN B_k AND C_l.

Figure 3-5 :

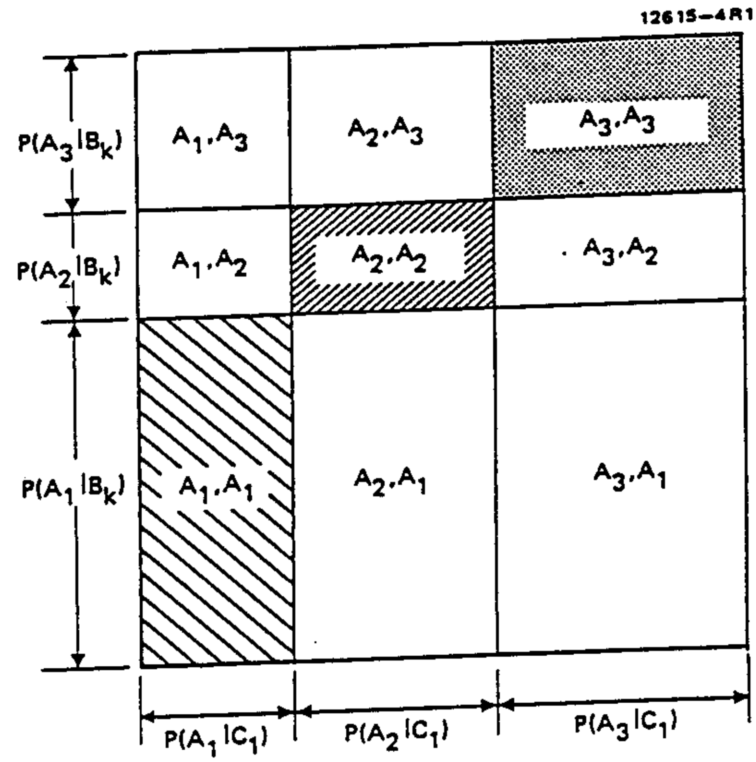
Approximation of Conditional Tensor - Trinary variable when dominance relation holds.



$$P(A_1 \mid B_i, C_j) = \alpha P(A_1 \mid B_i) \cdot P(A_1 \mid C_j)$$
$$P(A_2 \mid B_i, C_j) = \alpha P(A_2 \mid B_i) \cdot P(A_2 \mid C_j)$$

Figure 3-6 :

Approximation of Conditional Tensor - Binary variable when no dominance relation holds.



- DOMINANCE RELATION DOES NOT HOLD
- DIAGONAL ELEMENTS DETERMINE CONDITIONAL BELIEF DISTRIBUTION

Figure 3-7

Approximation of Conditional Tensor - Trinary variable when no dominance relation holds.

3.7 NUMERICAL EXAMPLES

To illustrate our inference scheme, this section containes a few numerical examples. Assume that Mr. Holmes himself heard the ALARM and is considering BUGRLARY (B and not B) as a possible cause of the ALARM (A and not A), as in Figure 3-8. Also assume that his a priori belief on BURGLARY $P(B|B^{+})$ is Ø.1 and the conditional probabilities of ALARM given BURGLARY is as follows.

$$P(A|B) = 0.7$$

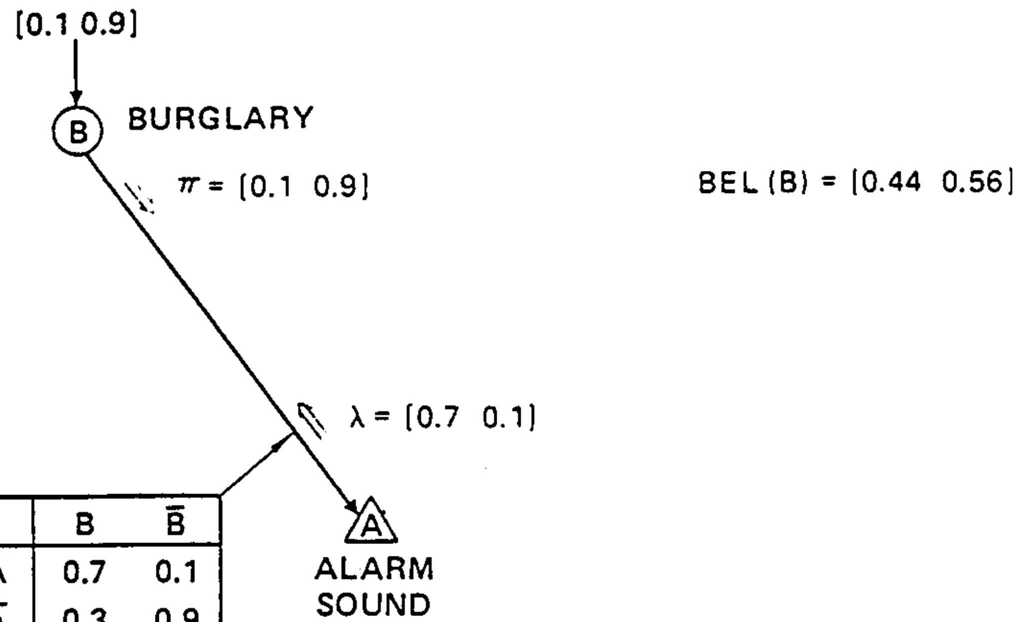
$$P(A|\text{ not } B) = 0.1$$

His posterioi belief on BURGLARY, $\underline{P}(B|B^{+},A)$, after hearing the ALARM SOUND is calculated by:

$$\begin{aligned}\underline{P}(B|B^{+},A) &= \underline{P}(A|B)\ \underline{P}(B|B^{+}) \\ &= [\ 0.7*0.1,\ 0.1*0.9] \\ &= [\ 0.07,\ 0.09] \\ &= [\ 0.4375\ 0.5625].\end{aligned}$$

The computation can be validated by applying Bayes's rule straightforward.

However, when Mr. Holmes thought of EARTHQUAKE (E and not E) as another potential cause with its prior



	B	$\bar{B}$
A	0.7	0.1
$\bar{A}$	0.3	0.9

Figure 3-8 : Computation of belief of BURGLARY when ALARM SOUND was heard.

distribution [0.2, 0.8], and with the conditional probabilities of:

$P(A|E) = 0.2$

$P(A|\text{ not } E) = 0.1,$

we need the conditional probability matrix P(ALARM|BURGLARY and EARTHQUAKE) to determine his belief distribution. This conditional probability tensor can be computed from the approximation formula suggested in the previous section. Since the state ALARM(A) dominates its complemetary state(not A), we have:

$$P(A|B,E) = [0.2 + 0.7*0.8,\ 0.3*0.8] = [0.76,\ 0.24]$$

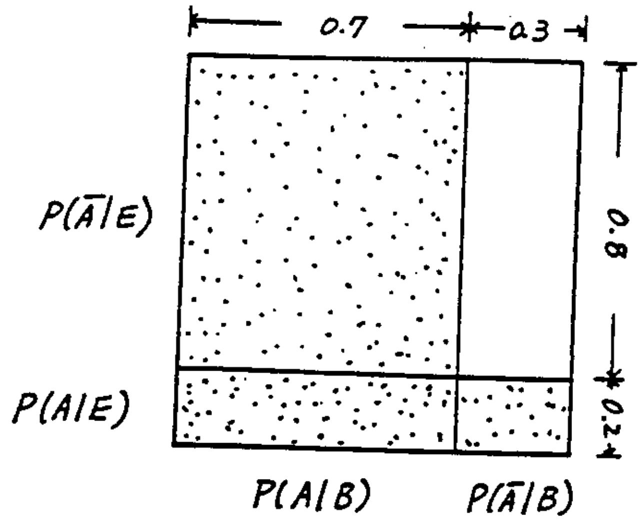
$$P(A|\text{not } B,\ E) = [0.2 + 0.1*0.8,\ 0.9*0.8] = [0.28,\ 0.72]$$

$$P(A|B,\ \text{not } E) = [0.1 + 0.7*0.9,\ 0.3*0.9] = [0.73,\ 0.27]$$

$$P(A|\text{not } B,\ \text{not } E) = [0.1 + 0.1*0.9,\ 0.9*0.9] = [0.19,\ 0.81].$$

See Figure 3-9. Thus, the belief of the occurrence of BURGLARY is reduced to 0.282, as shown in Figure 3-10, due to the listing of another potential cause of the ALARM SOUND,

Obtaining a supporting evidence, RADIO BROADCAST (R or not R), for the occurrence of EARTHQUAKE, with the conditional probabilities:



$$P(A|B,E) = 0.7 * 0.8 + 0.7 * 0.2 + 0.3 * 0.2$$
$$= 0.76$$
$$P(\bar{A}|B,E) = 0.3 * 0.8$$
$$= 0.24$$

Figure 3-9 : Approximation of probability of ALARM given BURGLARY and EARTHQUAKE.

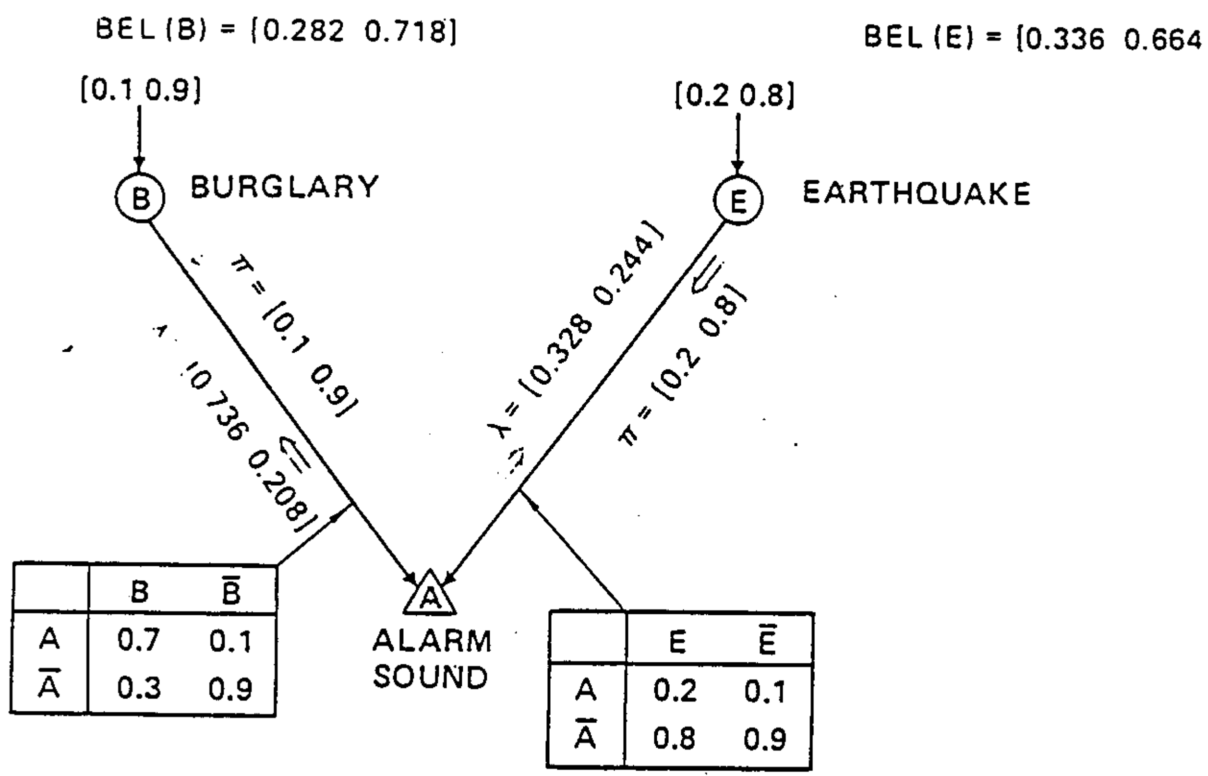


Figure 3-10: Computation of belief of BURGLARY after remembering EARTHQUAKE as another cause of the ALARM SOUND.

$P(R|E) = .8$

$P(R|\text{not } E) = 0.001,$

further reduces the belief in the occurrence of BURGLARY to 0.235, as shown as Figure 3-11.

3.8 AN ALTERNATIVE FORMULATION OF BELIEF PARAMETERS

In previous sections, we have defined belief parameters associated with the links of the Chow Tree. Similarly, we can also define belief parameters associated with the nodes of the Chow Tree. Consider again the Chow Tree shown in Figure 3-1. The current belief in a variable A is written as in Eq. (3-12):

$$(3\text{-}34) \quad \mathrm{BEL}(A_i) = P(A_i | D^{+}_{BA}, D^{+}_{CA}, D^{-}_{AX}, D^{-}_{AY}).$$

Let the belief committed to the variable A, considering just D^{+}_{BA}, be written with a subscript as:

$$(3\text{-}35) \quad \mathrm{BEL}_B(A_i) = P(A_i | D^{+}_{BA}).$$

Let the belief in variable A, considering all the data except D^{+}_{BA}, be written with a negative subscript as $\mathrm{BEL}_{-B}(A)$. For the Chow Tree of Figure 3-1, we can write:

$$(3\text{-}36) \quad \mathrm{BEL}(A) = \mathrm{BEL}_{BCXY}(A)$$

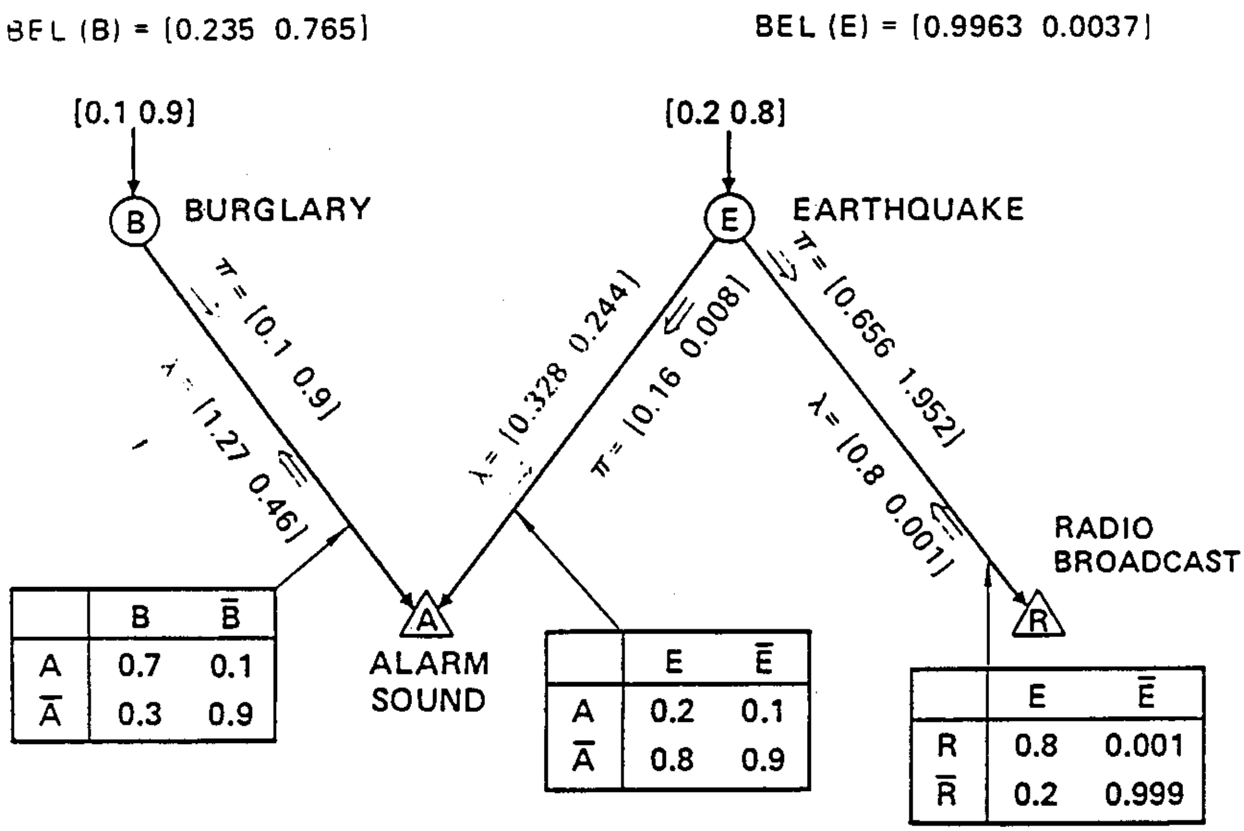


Figure 3-11 : Computation of belief of BURGLARY after observing a supporting evidence of EARTHQUAKE.

and

(3-37) $\mathbf{BEL}_{-B}(A) = \mathbf{BEL}_{CXY}(A)$.

We write $P(D^{-}_{AX}, D^{-}_{AY}|A_i)$, the probability of the occurrence of the data D^{-}_{AX} and D^{-}_{AY}, given the state A_i of a variable A, as $\mathbf{NLAMBDA}_{XY}(A)$:[5]

(3-38) $\mathbf{NLAMBDA}_{XY}(A_i) = P(D^{-}_{AX}, D^{-}_{A,Y}|A_i)$

and

(3-39) $\underline{\mathbf{NLAMBDA}}_{XY}(A) = (\mathbf{NLAMBDA}(A_1),$ --

The probability of the occurence of all the diagnostically influencing variables given in A_i is represented without subscript as $\mathbf{NLAMBDA}(A_i)$. We may write the probability of the occurence of all the diagnostically influencing variables except D^{-}_{XA}, given A_i as $\mathbf{NLAMBDA}_{-X}(A_i)$. For the Chow Tree of Figure 3-1, we may write:

(3-40) $\mathbf{NLAMBDA}(A) = \mathbf{NLAMBDA}_{XY}(A)$

because D^{-}_{XA} and D^{-}_{YA} constitute all the diagnostically influencing variables to A. Also we may write:

(3-41) $\mathbf{NLAMBDA}_{-X}(A) = \mathbf{NLAMBDA}_{Y}(A)$.

----

5. NLAMBDA connotes node-LAMBDA.

Similarly, we will write $P(A_i|D^+_{BA},D^+_{CA})$, the probability of A_i, given all the causally influencing data in D^+_{BA} and D^+_{CA}, as $\mathbf{NPI}_{BC}(A_i)$:

$$(3\text{-}42) \quad \mathbf{NPI}_{BC}(A_i) = P(A_i|D^+_{BA},D^+_{CA}).$$

The probability of A_i, given all the causally influencing data, is written without subscript as $\mathbf{NPI}(A_i)$; the probability of A_i, given all causally influencing data except D^+_{BA}, is written as $\mathbf{NPI}_{-B}(A_i)$. For the Chow Tree of Figure 3-1, we may write:

$$(3\text{-}43) \quad \mathbf{NPI}(A) = \mathbf{NPI}_{BC}(A)$$

and

$$(3\text{-}44) \quad \mathbf{NPI}_{-B}(A) = \mathbf{NPI}_{C}(A).$$

Relationships among **BEL**, **LAMNDA**, **PI**, **NLAMBDA** and **NPI**.

We can derive the following relations from the preceding definitions:

1. $\mathbf{BEL}(A) = \mathbf{NPI}(A)\ \mathbf{NLAMBDA}(A)$

2. $\mathbf{BEL}(A) = \mathbf{PI}_X(A)\ \mathbf{LAMBDA}_X(A)$, where X is a direct successor of A.

3. $\mathbf{BEL}_{-X}(A) = \mathbf{PI}_X(A)$

4. $\mathbf{BEL}_B(A) = \mathbf{NPI}_B(A)$, if B is a direct predecessor of A.

5. $\mathbf{BEL}_X(A) = \mathbf{NLAMBDA}_X(A)$, if X is a direct successor of A.

6. $\mathbf{NPI}_B(A) = \sum_i P(A|B_i)\mathbf{BEL}_{-A}(B_i)$, if B is a direct succcessor of A.

The fourth and fifth relations yield that:

1. $\mathbf{NPI}_B(A) = \underline{1}$ if B is not a causally influencing variable, namely, when no anticipatory support is committed to A through B.

2. $\mathbf{NLAMBDA}_X(A) = \underline{1}$ if X is not a diagnostically influencing variable, namely, when no evidential support is committed to A through X.

The last relation is because:

$$
\begin{aligned}
\text{(3-45)}\quad \mathbf{NPI}_B(A) &= \underline{P}(A|D^+_{BA}) \\
&= \sum_i \underline{P}(A|B_i)\ P(B_i|D^+_{BA}) \\
&= \sum_i \underline{P}(A|B_i)\ \mathbf{BEL}_{-A}(B_i) \\
&= \sum_i \underline{P}(A|B_i)\ \mathbf{PI}_A(B_i).
\end{aligned}
$$

Interpretation of the Independence Relations

The independence relations defined in the previous section may be interpreted in terms of BEL, NPI and NLAMBDA as follows:

1. Variable A and B are inter-causally independent iff:

$$NPI(AB) = NPI(A)NPI(B).$$

2. Variable A and B are inter-symtomically independent iff:

$$NLAMBDA(AB) = NLAMBDA(A)NLAMBDA(B).$$

3. Variable A and B are totally independent iff:

$$BEL(AB) = BEL(A)BEL(B)$$

i.e.,

$$NPI(AB) = NPI(A)NPI(B)$$

and

$$NPI(AB) = NLAMBDA(A)NLMABDA(B).$$

Let B and C be direct predecessors of A (see Figure 3-1). The anticipatory support from B to A, $NPI_B(A)$, and the anticipatory support from C to A, $NPI_C(A)$, are combined into a total anticipatory support from B and C, $NPI_{BC}(A)$. Let us introduce an operator, $\oplus$, which combines two anticipatory supports. We then write $NPI_{BC}(A)$, with the operator $\oplus$, as:

$$(3\text{-}46) \quad NPI_{BC}(A) = NPI_B(A) \oplus NPI_C(A).$$

By definition of NPI and by the structural assumption:

$$(3\text{-}47) \quad NPI_{BC}(A) = \underline{P}(A|D^+_{BA}, D^+_{CA})$$
$$= \sum_{ij} P(A|B_iC_j)P(B_iC_j|D^+_{BA}, D^+_{CA})$$

and

$$(3\text{-}48) \quad P(B_iC_j|D^+_{BA}, D^+_{CA}) = BEL_{-A}(B_iC_j)$$
$$= NPI_{-A}(BC)NLAMBDA_{-A}(BC)$$
$$= NPI(BC)NLAMBDA_{-A}(BC)$$
$$= NPI(B_i)NPI(C_j)NLAMBDA_{-A}(BC).$$

Because variables B and C are diagnostically independent if A is excluded:

$$NLAMBDA_{-A}(BC) = NLAMBDA_{-A}(B)NLAMBDA_{-A}(C)$$

Therefore,

$$(3\text{-}49) \quad NPI_B(A) \oplus NPI_C(A) = \sum P(A|B_iC_j)NPI(B_i)$$
$$NLAMBDA_{-A}(B)NPI(C_j)NLAMBDA_{-A}(C_j).$$

Since the product of $NPI(B_i)$ and $NLAMBDA_{-A}(B_i)$ is: $BEL_{-A}(B)$,

$$(3\text{-}50) \quad NPI_B(A) \oplus NPI_C(A) = \sum P(A|B_iC_j)BEL_{-A}(B)BEL_{-A}(C)$$

Also since $\mathbf{BEL}_A(B)$ is $\mathbf{PI}_A(B)$, we can write:

$$(3\text{-}51)\quad \mathbf{NPI}_B(A) + \mathbf{NPI}_C(A) = \Sigma\ P(A|B_iC_j)\mathbf{PI}_A(B)\mathbf{PI}_A(C)$$

Equation (3-51) reveals that the combining impacts of two direct predecessors of a node needs both the belief in the each predecessor without the data from the node and the conditional probability distribution of A given the two predecessors.

Let X and Y be direct successors of A (see Figure 3-1). The evidential support from X to A, $\mathbf{NLAMBDA}_X(A)$, and the evidential support from Y to A, $\mathbf{NLAMBDA}_Y(A)$, are combined into a total evidential support from X and Y, $\mathbf{NLAMBDA}_{XY}(A)$. By definition, the total evidential support can be written:

$$\begin{aligned}(3\text{-}52)\quad \mathbf{NLAMBDA}_{XY}(A) &= P(D^-_{AX}, D^-_{AY} \mid A)\\ &= P(D^-_{AX}|A)\ P(D^-_{AY} \mid A)\\ &= \mathbf{NLAMBDA}_X(A)\ \mathbf{NLAMBDA}_Y(A)\end{aligned}$$

The combined evidential support is simply the product of its components.

Note that the combining operators, $\oplus$ for NPI and the product for **NLAMBDA**, are commutative and associative: the order in which two operands are considered does not affect the result, while the manner of associating the factors into pairs for combining would not affect the final result.

$$\begin{aligned} NPI_B(A) \oplus NPI_C(A) \oplus NPI_D(A) &= NPI_C(A) \oplus NPI_B(A) \oplus NPI_D(A) \\ &= (NPI_B(A) \oplus NPI_C(A)) \oplus NPI_D(A) \\ &= NPI_B(A) \oplus (NPI_C(A) \oplus NPI_D(A)) \end{aligned}$$

These chracteristics of the operators are very pleasing. A new data source may be combined into existing ones without recalculating the impacts from the existing data sources.

Thus, the total strength of belief in A, BEL(A), is:

$$\text{(3-53)} \quad BEL(A) = \sum_{B^1,\ldots,B^n} P(A|B^1, \ldots, B^n) \left[\prod_i BEL_{-A}(B^i) \right] \prod_j NLAMBDA_X{}^j(A)$$

where $B^1, \ldots, B^n$ are n predecessors of A and $X^1, \ldots, X^m$ are m successors of A. Equation (3-53) suggests that the probability distribution of every variable in the network could be computed if the node corresponding to that variable contains two kinds of belief parameters:

1. $BEL_{-A}(B^i)$ for each of its predecessor node B^i.

2. $NLAMBDA_X{}^j(A)$ for each of its successor node X^j.

In this section, belief parameters have been described in terms of link parameters and in terms of node parameters. Belief propagation rules may be derived with either kind of parameters.

Chapter 4

SYSTEM ARCHITECTURE

4.1 OVERVIEW

CONVINCE assists a decision-making user in structuring his own knowledge for a rational assessment of the likelihood of uncertain events. It operates in an interactive mode in that the user's perception of the factors surrounding these events is elicited through an interviewing dialogue conducted in stylized English. The dialogue starts with CONVINCE's query about the user's main concern, i.e., a target hypothesis. CONVINCE then queries the user regarding relevant variables, available evidence and their impacts on that target variable. The overall impact of these variables and evidence is computed and the likelihood of the target hypothesis is updated. A variable whose analysis would be the most beneficial to resolve the uncertainties involving the target hypothesis is selected and the user's attention is focused on that variable. Then the query process is repeated until either no more information will contribute significantly or the user wants to terminate. The user can override the system's initiative at any time and provide information voluntarily.

CONVINCE performs their fuctions by using four component modules:

1. Elicitation module

2. Dialogue Controller

3. Inference Engine

4. Network Editor.

The elicitation module paraphrases queries and interprets the user's responses. The dialogue controller selects, at each step, the variable which potentially can contribute the most information to the target hypothesis. The inference engine produces an overall inference by computing the likelihood of a target hypothesis integrating all the evidence so far gathered. The network editor provides the user with options to modify the existing network, and accepts the information volunteered by the user. Data-flows among these component modules are shown in Figure 4-1.

CONVINCE has been designed as a stand-alone system, but it can also be used as a sub-system of a larger decision

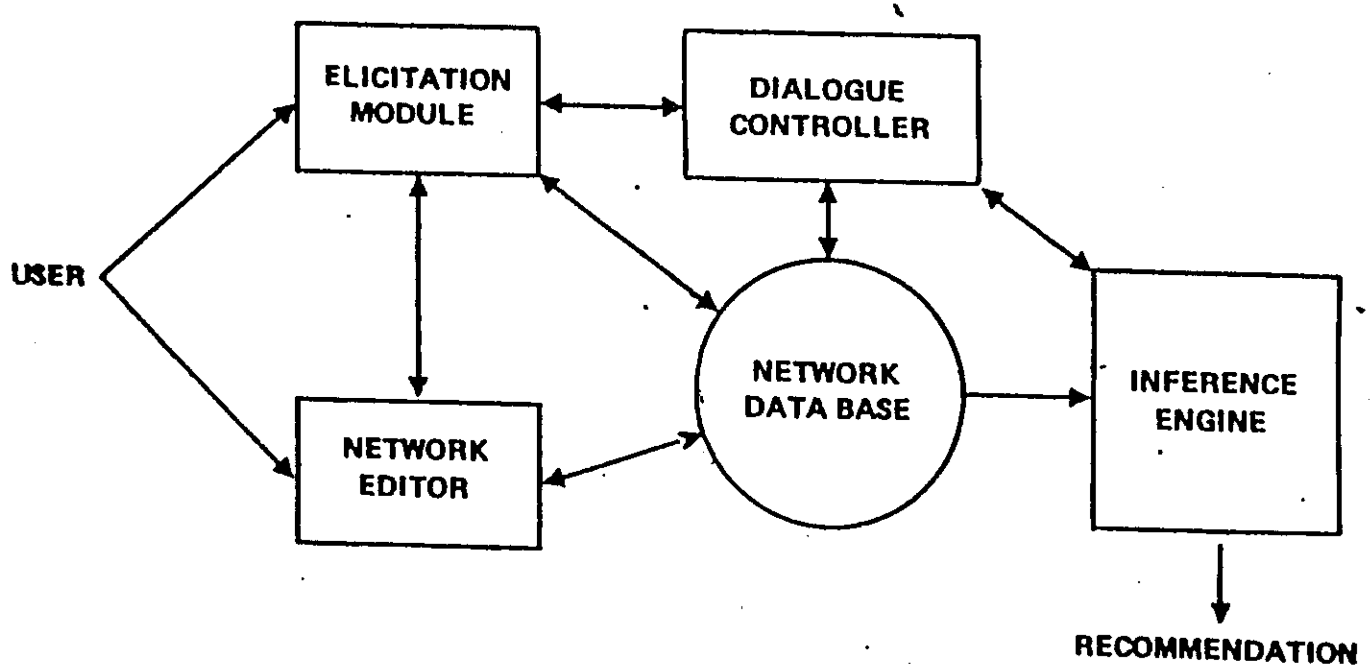


Figure 4-1 : System Components

support system such as GODDESS [Pearl 82]. When a refinement of the subjective assessment of the likelihoods of uncertain events is required, it could be called upon to provide more reliable assessments.

The system also can be used as an automated knowledge acquisition tool for constructing coarse-grain knowledge-based expert systems. The structure explicated and formalized in an interview session with a domain expert can be saved as a knowledge-base so that when a similar problem arises, the same inference engine and knowledge-base can be utilized to solve it.

4.2 ELICITATION MODULE

The elicitation module is responsible for the management of the interviews. This module guides the cyclic process of refining the information about variables and relations pertaining to the target hypotheses. The major information items regarding each variable can be placed in two categories: elicitable items and computable items. As these names imply, the elicitable items are to be elicited from the user, the computable items computed from items passed from their nearest neighbors. The elicitable items include the following:

- Variable description: A sequence of words that identifies and describes a variable and its meaning.

- Variable states: Names of mutually exclusive and exhaustive states that the variable may admit.

- Time of event occurrences: Times used by CONVINCE to determine the tense of its query phrases and to determine whether the existence of manifestational evidence should be queried. For example, if an event is declared as a future event, CONVINCE understands that no observed diagnostic evidence is available for the event.

- Role of variable in the information system: CONVINCE asks whether that variable is a data node or an intervening node whenever a new variable is introduced. These types of nodes are treated differently.

- Impacts from unexplicated causal data: Variables for which there are no predecessors, elicit an external parameter which summarizes their impact from an unexplicated background information. When the variable is linked later with its predecessor, this information is discarded.

- Dominance relationship: CONVINCE inquires whether dominance relationships exist among the states of the corresponding variables if a node has more than one causal factor. Probabilities conditioned on the causal factors are computed according to these relationships.

On the other hand, the computable items of a node include:

- Current state of belief: CONVINCE always updates and maintains all belief distributions upon acquiring new pieces of information.

- Probability tensor conditioned on the causal factors: The probability tensor conditioned on the direct causal factors is approximated according to the formulae of Section 3.6, and saved if a node has more than one causal factor.

- Importance to the target node: CONVINCE computes the relative importance of each elicited node to reducing the uncertainty of the target node. A detailed discussion will be found in Section 4.3.

Information items characterizing a link are also placed in two categories: elicitable items and computable items. The elicitable information items for a link include:

- Causal factor variables (predecessors): Variables which precede in time and have direct causal relations to the current node.

- Diagnostic manifestation variables (successors): Variables which succeed in time and are caused by the current node.

- Conditional probabilities relating neighboring variables: Conditional probabilities quantifying the links to predecessors and successors. The entries are normalized.

The computable items include:

- Current strength of causal evidence: PI, as defined in Section 3.3.

- Current strength of diagnostic evidence: LAMBDA, as defined in Section 3.3.

In general, all the necessary information is elicited through a prescribed sequence of interactions in the system-guided mode. Once this information is elicited, the node under analysis, called the current node, is said to have been "expanded". The flowchart in Figure 4-2 shows the complete elicitation procedure in a compact form and

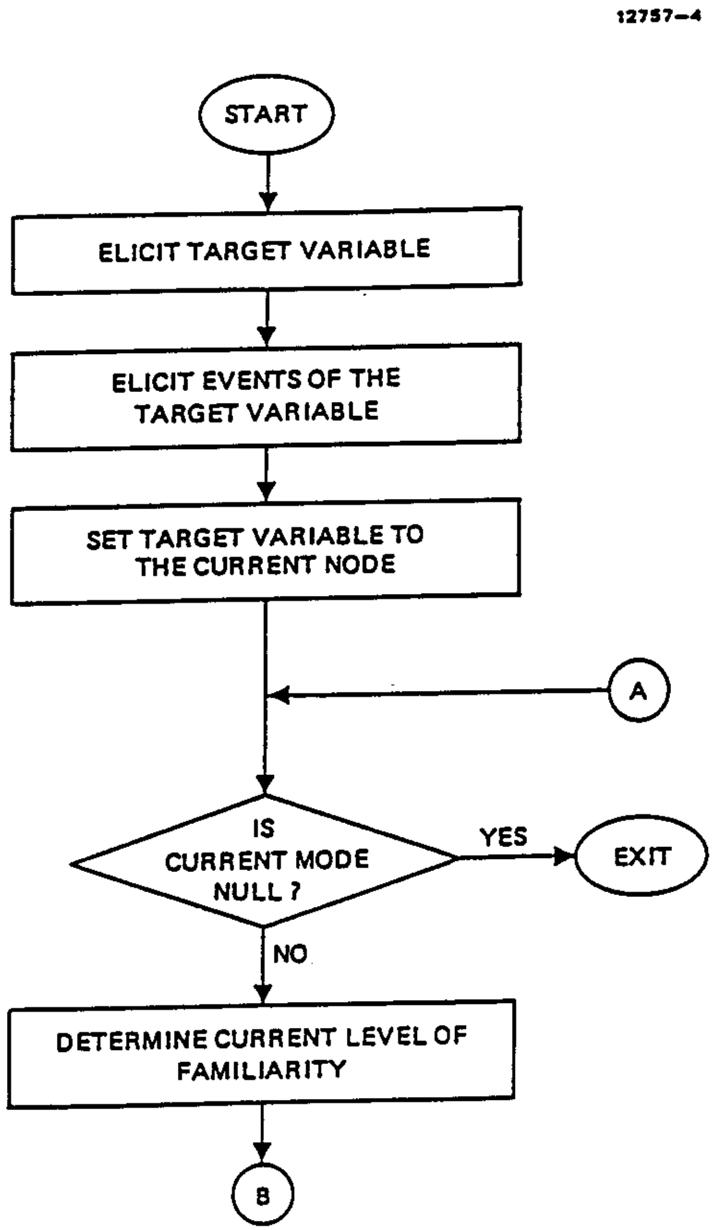


Figure 4-2 : Flow of the Elicitation Algorithm (cont'd)

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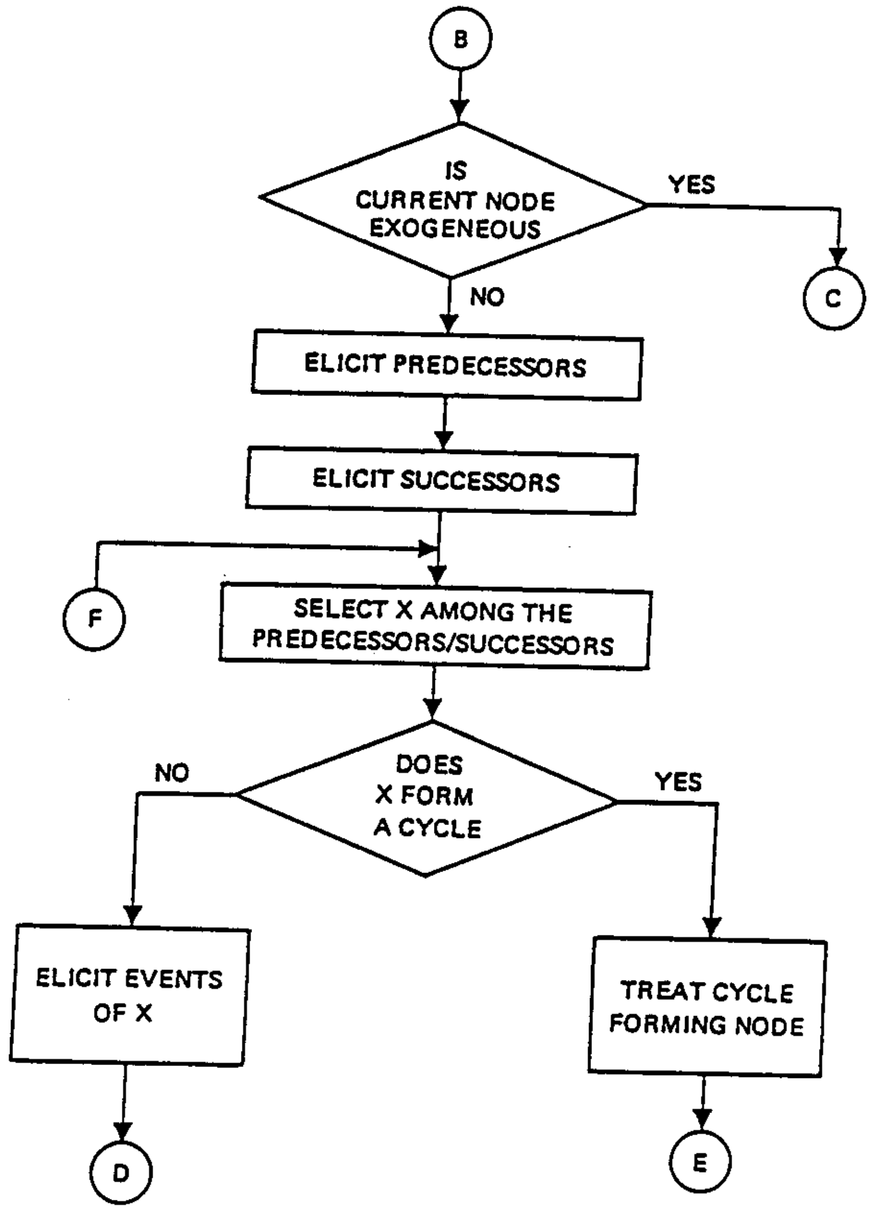
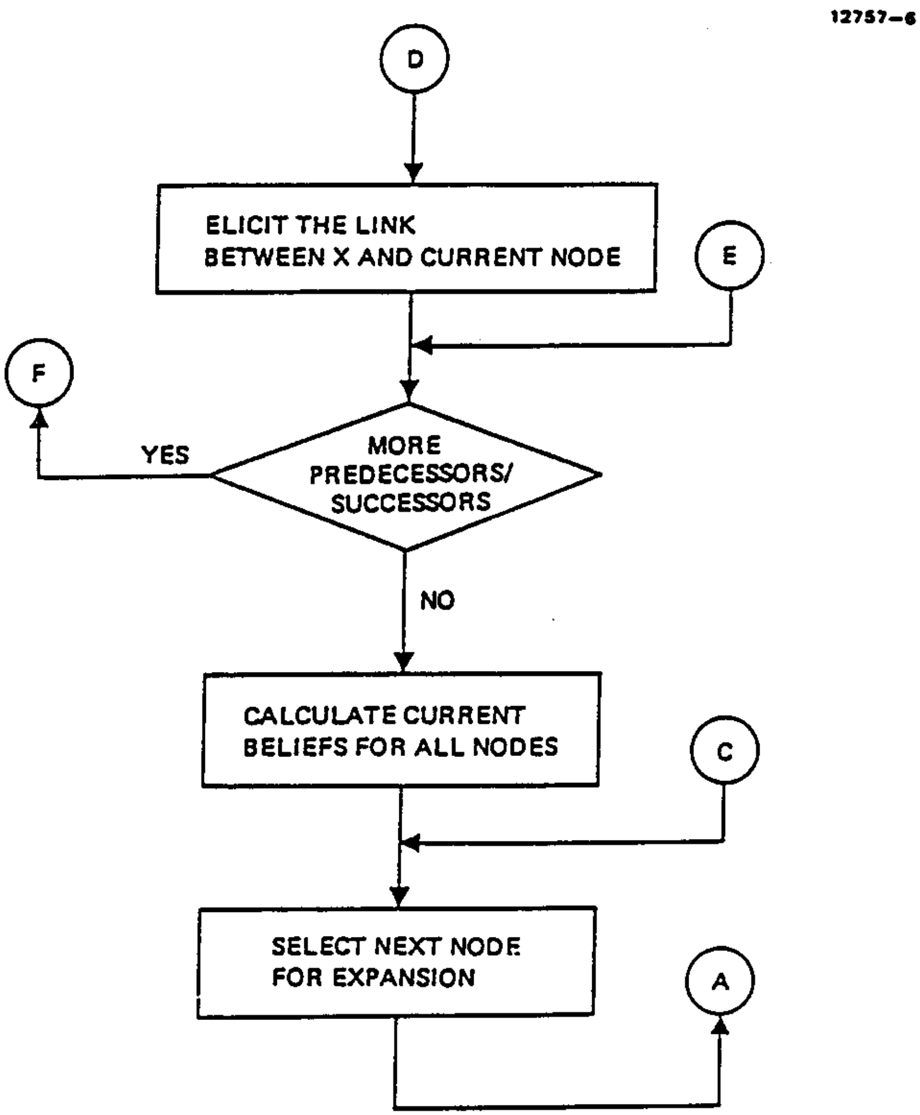


(cont'd)


12757-6
D
ELICIT THE LINK
BETWEEN X AND CURRENT NODE
E
F
YES
MORE
PREDECESSORS/
SUCCESSORS
NO
CALCULATE CURRENT
BELIEFS FOR ALL NODES
C
SELECT NEXT NODE
FOR EXPANSION
A

describes its overall logical structure.

CONVINCE has been programmed to understand several kinds of terminal characteristics such as screen-oriented input/output, reverse tone, and highlighting words by blinking and/or different tones. The current implementation understands the characteristics of VT1ØØ and ADM3 terminals.

The query phrases generated by CONVINCE are dependent on the user's familiarity with the system. Normally, CONVINCE addresses new users with with elaborate phrases, but uses terse phrases with experienced users. For this reason, a query about the user's familarity with CONVINCE is the very first question. The user's response is mapped into five discrete levels; each level produces queries at different levels of elaboration.

CONVINCE may change the degree of elaboration or style anytime within an interview session whenever the user shows he does not understand the query clearly by typing a question mark(?). Conversely, query phrases becomes terse as time passes by because after a type of query has been asked at a particular level of elaboration a number of times, the phrasing becomes less elaborate.

For each level of elaboration, a number of query patterns exist, while for each query pattern, a number of substitutable synonyms are available. The selection of a pattern or a synonym is done at random according to a distribution designated by the system designer. For this reason, the chance of generating the same query sentence twice in a row is very small even when the number of stored patterns and synonyms is not large. As an example, a query pattern is shown in Figure 4-3(a). Query patterns and synonyms are selected in a uniform random distribution if the first element of the list is a ??. If the first element is a ???, then patterns or synonyms are selected according to the distribution designated by their second element. Query phrases generated from the query pattern of Figure 4-3(a) are shown in Figure 4-3(b).

Whenever CONVINCE generates a query, it also generates the user's expected responses. For example, probabilities should be quantified by a number between Ø and 1; yes/no questions should expect only yes/no answers. If the user's response does not match the system's expectation, a warning message is produced and the user is prompted again.

CONVINCE provides a number of utility functions for friendly user interactions. These include HELP utilities, SHOW utilities, SYSTEM utility, and INFERENCE utility. The

```
[(?? (WHAT (?? FACTORS CAUSES (TYPE OF DEVELOPMENT))
          (?? COULD MIGHT WOULD)
          HAVE
          (?? INFLUENCED (LED TO)
              (ACTED ON)
              (BROUGHT ABOUT)
              PROMOTED CAUSED IMPACTED CONTRIBUTED
              INDUCED FURTHERED)
          (?? (THE (?? OCCURRENCE HAPPENING REALIZATION)
                   OF
                   (?? THE (THE ABOVE)))
              (?? THE (THE ABOVE)))
          (?? EVENTS STATES POSSIBILITIES SITUATIONS)
          (?? (CONNECTED WITH)
              OF)
          $FNODE ?)
    ((?? WOULD COULD CAN)
     YOU
     (?? ENTER LIST TYPE GIVE)
     (?? FACTORS CAUSES (TYPE OF DEVELOPMENT))
     THAT
     (?? COULD MIGHT WOULD)
     HAVE
     (?? INFLUENCED (LED TO)
         (ACTED ON)
         (BROUGHT ABOUT)
         PROMOTED FURTHERED CAUSED IMPACTED CONTRIBUTED
         INDUCED)
     ((?? (THE (?? OCCURRENCE HAPPENING REALIZATION)
               OF
               (?? THE (THE ABOVE)))
          (?? THE (THE ABOVE)))
      (?? EVENTS STATES POSSIBILITES SITUATIONS)
      (?? (CONNECTED WITH)
          OF)
      $FNODE "?"])
```

Figure 4-3 (a) A pattern of query phrase

- What factors have caused the occurrence of the events of the OUTCOME OF 1984 ELECTION ?

- Would you enter type of develpment that could have impacted the situations of the OUTCOME OF 1984 ELECTION ?

Figure 4-3 (b) : Query sentences generated from the pattern in Figure 4-3 (a) when $fnode has been bound to (OUTCOME OF 1984 ELECTION).

HELP utilities provide information about the available commands, SHOW utilities display the current network structure in stylized formats, and SYSTEM utility provides the means for interfacing CONVINCE with the host programming system, currently with the INTERLISP system. This is a handy debugging tool in the developmental stage. The INFERENCE utility permits the user to examine the current beliefs of the various states of the target variable at any time.

An interview may be interrupted at any point and resumed at that point in a later session. This means that a structure elicited in one session can be saved and retrieved in a later session. Elicited structures are indexed and cataloged with the help of the host operating system. In this manner, a knowledge base can be constructed for the analysis of specific situations. Should a similar situation arise at a later time, the structure can be recalled and used as a knowledge source.

The network editor, designed as a complementary tool for the elicitation module, provides the user with several options such as voluntary input and direct modification of the network structure. The use of the network editor requires a high degree of familiarity with CONVINCE's representational machanism and internal data structures.

However, experienced users should find the network editor a useful tool for constructing causal networks because it allows navigating, inspecting and modifying the structure freely. Once the network editor is invoked, maintaining structural consistency becomes the user's responsibility, although the system helps the user in this task.

4.3 DIALOGUE CONTROLLER

The explication of the causality network is initiated by characterizing a target variable and searching for its relevant variables and relations. Once a variable has been examined, - its admitted events listed, its causal and diagnostic variables elicited, and its links from or to the variable quantified, - we say that the variable has been "expanded." Then a new variable is selected and its relevant information is searched. This mode of incremental network generation corresponds to the expansion of the network outward from the target node. The growth of the network is governed by sensitivity considerations.

The search for new information is an important component in many AI systems and is known as control strategy. Well-known control strategies include breadth-first, depth-first and heuristic search strategy.

The breadth-first strategy explores information layer by layer, so that a shallow node will be explored before a deep node. The depth-first strategy explores information through a path until it reaches a dead end, one where no more information is available at the node. The breadth-first and depth-first algorithms are "blind" search methods in the sense that information about the goal state is not utilized in the determination of the exploration sequences. By contrast, the heuristic search algorithm utilizes information contained in the goal states to determine expansion sequences.

The control strategy adopted by CONVINCE is a heuristic search procedure which, at any given time, selects the most "beneficial" node for next expansion. The "benefit" of a node is defined in terms of the change of likelihood anticipated from expanding it. More formally, the benefit of a node is defined by the mean distance between the prior probability distribution of the target node before the expansion of the node, and the posterior probability distribution of the target node after expansion of the node. Let P(T) be the prior probability distribution of the target node T before expanding node N, and $\underline{P}(T|N)$ be the posterior probability distribution after expanding node N. The benefit of expanding node N, B(N), is defined by:

(4-1) $B(N) = I(P(T|N), P(N))$

where I(,) is the closeness measure of two probability distributions defined by:

$$(4\text{-}2) \quad I(P, P_a) = \sum P(X_i) \log [P(X_i)/P_a(X_i)]$$

Among the candidate nodes, the node that yields the largest benefit will be selected for next expansion. This control strategy is very reasonable and appeals to our intuition.

Because the computation of Eq (4-1) is very time-consuming, the benefit measure of a node is approximated in CONVINCE by a heuristic combination of two scalar numbers representing the importance and the uncertainty of that node. The importance of a node is measured by its impact on the target variable, and its uncertainty by the well-known entropy function. Thus, the benefit of node N is approximated by:

$$(4\text{-}2) \quad B(N) = IMPORTANCE(N) * ENTROPY(N).$$

The IMPORTANCE of a son [1] to its parent node is measured by normalized mutual information, where the mutual information between two variable A and B is defined by

$$(4\text{-}3) \quad M(A_i,B_j) = \sum_{ij} P(A_i, B_j) \log (A_i, B_j)/P(A_i)P(B_j).$$

----

1. Node B is called a son of node A if node B is introduced by exploring node A. It should not be confused with the causality predecessor-successor relations.

Therefore, IMPORTANCE is a measure that yields a value between zero and one where large values represent a higher dependency. It can be computed recursively since the importance of a grandson to its grandparent is a product of its parent's importance to the grandparent with the node's importance to its parent.

$$\text{(4-4)} \quad \text{IMPORTANCE}(N') = \text{IMPORTANCE}(N) * M(N,N') \Big/ \sum_{N'' \in \text{son}} M(N,N'')$$

N' is a son of N and M(N,N') is the mutual information among N and N'. The IMPORTANCE of the target node is set to 1 as a basis for the recursive computational algorithm.

ENTROPY, which is defined as:

$$\text{(4-5)} \quad \text{ENTROPY}(N) = \sum P(N_i) \log P(N_i),$$

is frequently used for measuring uncertainties in information theory. The mutiplication of IMPORTANCE by ENTROPY satisfies our desire to expand a node with a high uncertainty earlier than one with low. However, this provision produces an unpleasant effect when a data node waits to be expanded. A node observed to have a zero entropy so that its calculated benefit is also zero will not be expanded until all other nodes have been. In the hierarchical tree representation scheme, no observed data node needs to be expanded since no further exploration over data nodes can affect target variable's likelihood. However, in the inference network adopted by CONVINCE, a variable may influence others through their common manifestation variable even after the state of the manifestation variable has been observed. Thus, even when a variable is observed with certainty, the exploration of its various causes is still needed. To remedy this unpleasant effect, CONVINCE expands any data node, if needed, prior to any unexpanded intervening node.

Chapter 5

AN EXAMPLE

The following hypothetical situation was chosen as an example for demonstrating the system's operation. Imagine a political analyst who wishes to forecast the outcome of the 1984 presidential election. Through the interviewing dialogue, the analyst's inference structure is elicited, and the likelihood of the target variable, OUTCOME OF 1984 ELECTION, will be computed. The causal relations of the hypothetical situation are shown in Figure 5-1.

The basic format for representing the interaction with CONVINCE follows. In the system-guided mode, each query generated by CONVINCE starts with a number in squared brackets. If the system expects the answer to consist of more than one item, it prompts the user's input with an item number in a pair of parentheses; otherwise it prompts with an empty pair of parentheses. When the system is waiting for an input for the entry of a table, the table is displayed and the place where the new entry will go is indicated by the cursor position. This terminal-dependent

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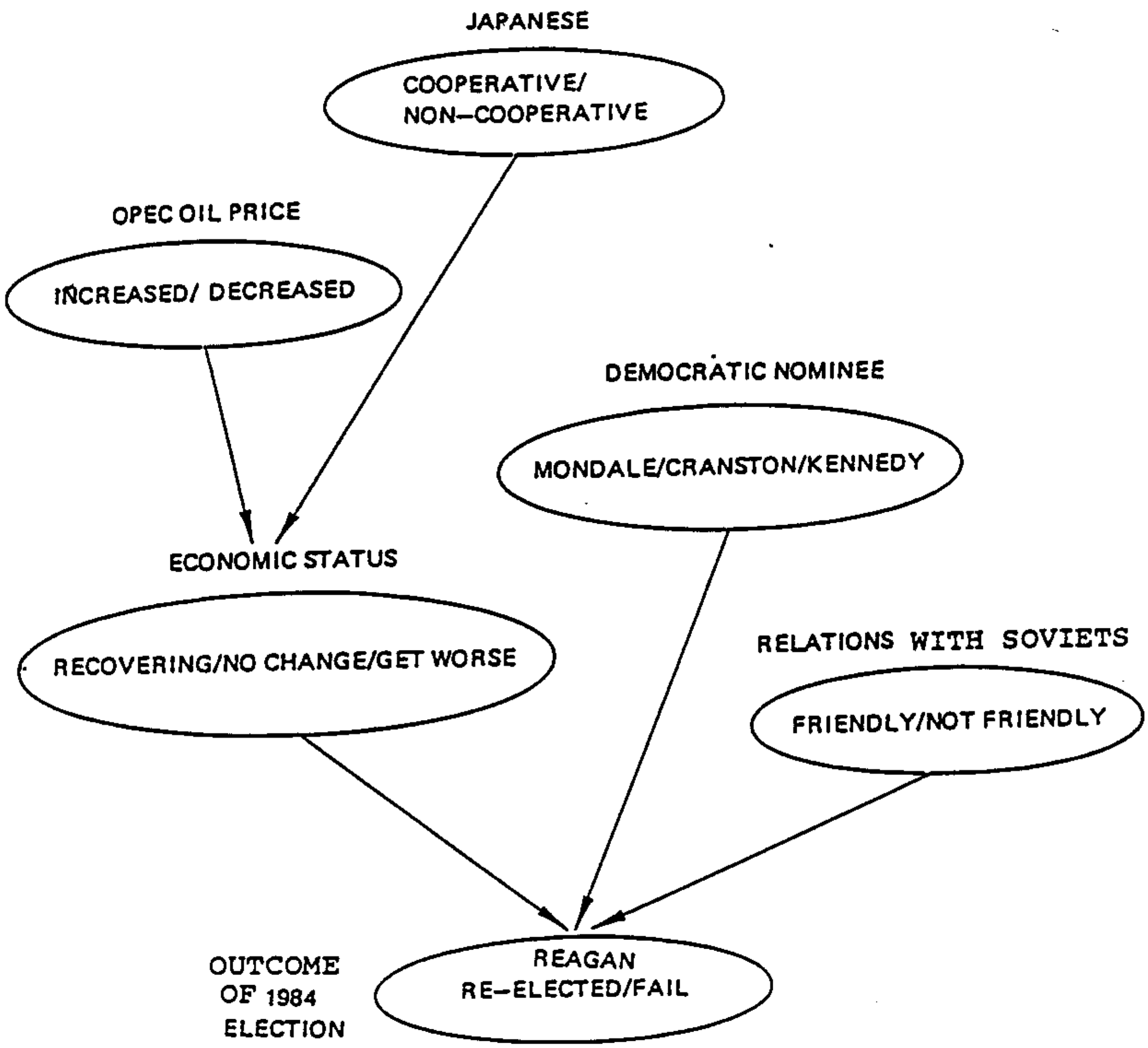

Figure 5-1 : Causal Relations of OUTCOME OF 1984 ELECTION dynamic input mode is not illustrated here because it needs too many snapshots of the display screen. Instead, the standard linear mode of interaction is shown in the following demonstration, where the input typed by the user is underscored to distinguish it from the system's output. The author's comments are indented and enclosed by angled brackets < >.

```
_(new)
                < CONVINCE is called. A new session begins >

Enter the terminal type : xxxx

                < CONVINCE understands the characteristics
                  of some terminals. If the current terminal
                  characteristics are known, the
                  screen-oriented input mode will be used
                  instead the standard linear mode.>

***********************************************************
*                        CONVINCE:                        *
*      A CONVersational INference Consolidation Engine    *
*                                                         *
***********************************************************

This session started at 8-Feb-83 16:17:53

[1] How many times have you used this system ?
    Select one:
    (1) It is the first time
    (2) less than 5 times
    (3) less than 10 times
    (4) 10 times or more

    Your answer ?
     ( ) 1

                < The user's familiarity with this system
                  has been questioned. Depending on his
                  response, CONVINCE chooses the appropriate
                  query paraphrases. >

[2] Please type a sentence or a phrase which describes the
```

eventuality that is your main concern.
( ) outcome of 1984 election

< The target-variable name has been elicited. This name is used to index the entire problem. >

[3] The possibilities involving the OUTCOME OF 1984 ELECTION

1: will happen. - Future Events
2: have happened. - Present/Past events

Select a number.
( ) 1

< The time of the events has been questioned. CONVINCE uses the time of the events to select the proper query phrases. >

[4] What values might the OUTCOME OF 1984 ELECTION assume ?
(1) ?

< The query phrase seems unclear to the user. He requests for an elaboration of this query by entering a question mark (?). >

[4] What are the various levels or events that can develope under the OUTCOME OF 1984 ELECTION ?
Enter one at a line.
(1) Reagan re-elected
(2) Reagan fail to be re-elected
(3) /

< The states of the target variable have been elicited. A slash(/) or a star (*) indicates no more items. >

[5] What factors can induce the above states connected with the OUTCOME OF 1984 ELECTION ?
(1) ?

[5] What types of developments may lead to the happening of the possibilities connected with the OUTCOME OF 1984 ELECTION ?
(1) ecomomic status
(2) democratic nominee
(3) (1)
...economic status
- changed
(3) relations with the Soviets
(4) /

< The causal variables influencing the target variables are elicited. Note that the user

was allowed to correct an error in a variable name.

Since the current node, the OUTCOME OF 1984 ELECTION, represents a future event, its manifestation states cannot be observed. Therefore, queries for eliciting its manifestation states are skipped. >

[6] The states or happenings involving the ECONOMIC STATUS

1: will happen. - Future Events.
2: have happened. - Present / Past Events

Select a number.
( ) 1

[7] What possibilities may happen under the ECONOMIC STATUS ?
Enter one by one.
(1) recovery
(2) worsening recession
(3) no change
(4) /

[8] To what degree do you believe that the OUTCOME OF 1984 ELECTION would be REAGAN RE-ELECTED if you knew that the ECONOMIC STATUS would be RECOVERY ? (Use a Ø to 1 scale for assessing your certainty or the strengh of your belief.)
( ) .9

[9] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the ECONOMIC STATUS is RECOVERY ? (Use a Ø to 1 scale.)
( ) .1

< The relationship between the ECONOMIC STATUS and the OUTCOME OF 1984 ELECTION is being elicited. These conditional probabilities are later normalized in order not to burden the user with this task.

These conditional probabilities can be elicited in a table format if the terminal's characteristics are known to the system, which greatly simplifies the interactions. >

[1Ø] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN RE-ELECTED given the ECONOMIC STATUS is WORSENING RECESSION ? (Use a Ø to 1 scale.)
( ) .4

[11] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the ECONOMIC STATUS is WORSENING RECESSION ? (Use a Ø to 1 scale.)
( ) .5

[12] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION is REAGAN RE-ELECTED given the ECONOMIC STATUS is NO CHANGE ? (Use a Ø to 1 scale.)
( ) .5

[13] Enter the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the ECONOMIC STATUS is NO CHANGE. (Use a Ø to 1 scale.)
( ) 4

Your answer is out of expectation. Try again.
( ) .4

< Although the numbers will be normalized, each should be in the range between Ø and 1. >

[14] Could you give me a ballpark estimate of the likelihood that the ECONOMIC STATUS would be RECOVERY. (Use a scale Ø to 1 for assessing your belief. The higher the value, the stronger the belief.)
( ) .4

[15] Please type the degree of belief in the ECONOMIC STATUS being GETTING WORSE.
( ) .3

[16] Please type the degree of belief in the ECONOMIC STATUS being NO CHANGE considering only the causal factors.
( ) .3

< The influence of unexplicated causal factors of the variable ECONOMIC STATUS has been elicited. These numbers will be discarded once the causal factors of the ECONOMIC STATUS are explicated. For more discussion, see Chapter 3. >

[17] The events involving the DEMOCRATIC NOMINEE

1: will happen - Future Events
2: have happened. - Present / Past Events.

Select a number.
( ) 1

< The relations between the target node and
the DEMOCRATIC NOMINEE is being explicated.
>

[18] What developments do you perceive the DEMOCRATIC NOMINEE will be ? Enter one by one.
(1) Mondale
(2) Cranston
(3) Kennedy
(4) /

[19] Assuming that you know the DEMOCRATIC NOMINEE is MONDALE, how strongly do you believe the OUTCOME OF 1984 ELECTION will be REAGAN RE-ELECTED ? (Use a scale of Ø to 1 for assessing your certainty or strength of your belief.)
( ) .4

[2Ø] Enter the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the DEMOCRATIC NOMINEE is MONDALE. (Use a Ø to 1 scale.)
( ) .3

[21] Enter the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN RE-ELECTED given the DEMOCRATIC NOMINEE is CRANSTON. (Use a Ø to 1 scale.)
( ) .4

[22] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the DEMOCRATIC NOMINEE is CRANSTON ? (Use a Ø to 1 scale.)
( ) .4

[23] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN RE-ELECTED given the DEMOCRATIC NOMINEE is KENNEDY ? (Use a Ø to 1 scale.)
( ) .3

[24] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the DEMOCRATIC NOMINEE is KENNEDY ? (Use a Ø to 1 scale.)
( ) .4

[25] Before we get into a deep analysis, give a crude estimate of your belief that the DEMOCRATIC NOMINEE would be MONDALE. Enter a number between zero and one. A higher number represents stronger beleif.
( ) .5

[26] Please give your belief about the DEMOCRATIC NOMINEE being CRANSTON considering only the factors that may cause this event.

( ) .4

[27] Enter the degree of belief about the DEMOCRATIC NOMINEE being KENNEDY considering only the factors that may cause this event.
( ) .1

[28] The possibilities of the RELATIONS WITH THE SOVIETS

1: will happen - Future Events
2: have happened - Present / Past Events

Select a number.
( ) 1

< The third causal factor of the target variable is being examined. >

[29] What levels or events may happen under the RELATIONS WITH THE SOVIETS ?
Enter one by one.
(1) friendly
(2) not friendly
(3) /

[30] Assuming that you knew the RELATIONS WITH SOVIETS is FRIENDLY, how strongly do you believe the OUTCOME OF 1984 ELECTION being REAGAN RE-ELECTED ? Give me a number between zero and one where the higher number represents more belief.
( ) .54

[31] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given the RELATIONS WITH THE SOVIETS is FRIENDLY ? (Use a 0 to 1 scale.)
( ) .46

[32] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN RE-ELECTED given the RELATIONS WITH THE SOVIETS is NOT FRIENDLY ? (Use a 0 to 1 scale.)
( ) .50

[33] What is your assessment of the likelihood of the OUTCOME OF 1984 ELECTION being REAGAN FAIL TO BE RE-ELECTED given THE RELATIONS WITH THE SOVIETS is NOT FRIENDLY ? (Use a 0 to 1 scale.)
( ) .50

[34] Before we get into a deep analysis, give a ballpark estimate of your belief that the RELATIONS WITH THE SOVIETS would be FRIENDLY. Give a number between zero

and one. A higher number represents stronger belief.
( ) .4

[35] Enter assessment of the belief that the RELATIONS WITH THE SOVIETS is NOT FRIENDLY.
( ) .6

[36] Suppose some factors cause the OUTCOME OF 1984 ELECTION to be REAGAN RE-ELECTED. Can I assume the OUTCOME OF 1984 ELECTION will be maintained even though the other factors tend to cause REAGAN FAIL TO BE RE-ELECTED ?

(1) Yes, that's true.
(2) No, the state becomes REAGAN FAIL TO BE RE-ELETED
(3) None of the above - I don't know

Select a number
( ) 3

< CONVINCE attempts to verify whether the dominance relationship holds with respect to this variable. This relationship will be used for the approximation of the high order conditional probability matrix given by the causes. Since the dominance relationship does not hold for this variable, the second method, ratio of diagonal elements, will be used. >

[37] What causes will contribute to the happening of the above events connected with the ECONOMIC STATUS ?
If you don't want to explore the happenings of the ECONOMIC STATUS any further, you may type $exo.
(1) OPEC oil price
(2) Japanese cooperation
(3) /

< A new cycle starts here where a new variable is selected for further exploration. The user has an option to override the system's selection. >

[38] The states or happenings of the OPEC OIL PRICE

1: will happen - Future Events
2: have happened - Present / Past Events

Select a number.
( ) 1

[39] How do you describe the various possibilities connected with the OPEC OIL PRICE ? Enter one in each line.
(1) increased
(2) decreased

(3) /

[40] Assuming that you knew the OPEC OIL PRICE is INCREASED, how strongly do you believe the ECONOMIC STATUS would be RECOVERY ? (Use a scale from Ø to 1 for assessing your certainty or the strength of your belief.)
( ) .3

[41] Enter the likelihood of the ECONOMIC STATUS being WORSENING RECESSION given the OPEC OIL PRICE is INCREASED. (Use a Ø to 1 scale.)
( ) .7

[42] What is your assessment of the likelihood of the ECONOMIC STATUS being NO CHANGE given the OPEC OIL PRICE is INCREASED ? (Use a Ø to 1 scale.)
( ) .5

[43] Enter the likelihood of the ECONOMIC STATUS being RECOVERY given the OPEC OIL PRICE is DECREASED. (Use a Ø to 1 scale.)
( ) .7

[44] What is your assessment of the likelihood of the ECONOMIC STATUS being WORSENING RECESSION given the OPEC OIL PRICE is DECREASED ? (Use a Ø to 1 scale.)
( ) .3

[45] What is your assessment of the likelihood of the ECONOMIC STATUS being NO CHANGE given the OPEC OIL PRICE is DECREASED ? (Use a Ø to 1 scale.)
( ) .3

[46] Would you give me a ballpark estimate of the likelihood that the OPEC OIL PRICE would be INCREASED ? Give a number between zero and one. A higher number represents stronger belief.
( ) .3

[47] Please give the degree of belief on the OPEC OIL PRICE being DECREASED considering only the factors that may cause this event.
( ) .7

[48] The developments of JAPANESE COOPERATION

1: will happen. - Future events
2: have happened. - Present / Past Events

Select a number.
( ) 1

[49] What names can be given to the development possibly to

happen under the JAPANESE COOPERATION ? Enter one by one.
(1) voluntarily cut their export to USA
(2) increase their exports to USA
(3) /

[5Ø] To what degree do you believe that the ECONOMIC STATUS would be RECOVERY if you knew that the JAPANESE COOPERATION was VOLUNTARILY CUT THEIR EXPORTS TO USA ? (Use a scale from Ø to 1 for assessing your certainty or the strength of your belief.)
( ) $show network

< The structure so far elicited will be displayed in a simple form. >

Node Id	Type	Description
node1-OO1E	T	OUTCOME OF 1984 ELECTION
node2-ES	A	ECONOMIC STATUS
node3-DN	A	DEMOCRATIC NOMINEE
node4-RWTS	A	RELATIONS WITH THE SOVIETS
node5-OOP	A	OPEC OIL PRICE
node6-JC	NIL	JAPANESE COOPERATION

Link Id	From	To	Quantified
link1	node2-ES	node1-OO1E	yes
link2	node3-DN	node1-OO1E	yes
link3	node4-RWTS	node1-OO1E	yes
link4	node5-OOP	node2-ES	yes
link5	node6-JC	node2-ES	no

< Note that the node names have been constructed by a sequence number followed by a mnemonic-id. CONVINCE constructs the mnemonic-id by concatenating the first characters of each word. The purpose of the mnemonic-ids is to help the user recall the meanings of variables. >

[5Ø] Assuming that you knew the JAPANESE COOPERATION is VOLUNTARILY CUT EXPORTS TO USA, how strongly do you believe the ECONOMIC STATUS would be RECOVERY ? (Use a scale from Ø to 1 for assessing your certainty or the strength of your belief.)
( ) $show link 1

< A particular link will be listed in a full format. >

```
------------------------------------------------------------
link 1
------------------------------------------------------------
From Node : node2 - ECONOMIC STATUS
To   Node : node1 - OUTCOME OF 1984 ELECTION

    A1 : (RECOVERY)
    A2 : (WORSENING RECESSION)
    A3 : (NO CHANGE)

    B1 : (REAGAN RE-ELECTED)
    B2 : (REAGAN FAIL TO BE RE-ELECTED)


        Normalized Conditional Belief of B given A
                I           I           I
                I     A1    I     A2    I     A3
                I           I           I
        ---------------------------------------------------
          B1    I    Ø.9    I    Ø.444  I    Ø.556
        ---------------------------------------------------
          B2    I    Ø.1    I    Ø.556  I    Ø.444
```

[5Ø] What is your assessment of the likelihood of the ECONOMIC STATUS being RECOVERY given the JAPANESE COOPERATION is VOLUNTARILY CUT EXPORTS TO USA ? (Use a Ø to 1 scale.)
( ) $show node 1

< A particular node will be dispalyed in a full format. >

```
------------------------------------------------------------
Node Id          : node1-OO1E
Node Type        : Target node
Description      : OUTCOME OF 1984 ELECTION
Predecessor      : link1-node2; link2-node3; link3-node4;
Successor        :
    Current belief          Event
     .6917Ø86              (REAGAN RE-ELECTED)
     .3Ø82914              (REAGAN FAIL TO BE RE-ELECTED)

Time of the events : Future
Relative Importance to the target node : 1.Ø
------------------------------------------------------------
```

[5Ø] Suppose you knew the JAPANESE COOPERATION is VOLUNTARILY CUT EXPORTS TO USA. What is your assessment of the likelihood of the ECONOMIC STATUS is RECOVERY ? (Use a scale from Ø to 1 for assessing your certainty or the strength of your belief.)
( ) .6

[51] Enter the likelihood of the ECONOMIC STATUS being NO CHANGE given the JAPANESE COOPERATION is VOLUNTARILY CUT EXPORTS TO USA. (Use a Ø to 1 scale.)
( ) .4

[52] Enter the likelihood of the ECONOMIC STATUS being WORSENING RECESSION given the JAPANESE COOPERATION is VOLUNTARILY CUT EXPORTS TO USA. (Use a Ø to 1 scale.)
( ) .4

[53] Enter the likelihood of the ECONOMIC STATUS being RECOVERY given the JAPANESE COOPERATION is INCREASE THEIR EXPORTS TO USA. (Use a Ø to 1 scale.)
( ) .3

[54] What is your assessment of the likelihood of the ECONOMIC STATUS being WORSENING RECESSION given the JAPANESE COOPERATION is INCREASE THEIR EXPORTS TO USA. (Use a Ø to 1 scale.)
( ) .3

[55] What is your assessment of the likelihood of the ECONOMIC STATUS being NO CHANGE given the JAPANESE COOPERATION is INCREASE THEIR EXPORTS TO USA. (Use a Ø to 1 scale)
( ) .3

[56] Before we get into a deep analysis, give a crude estimate of the likelihood that the JAPANESE COOPERATION would be VOLUNTARILY CUT THEIR EXPORTS TO USA ? (Use a Ø to 1 scale)
( ) Ø.7

[57] Type your belief of the JAPANESE COOPERATION would be INCREASE EXPORTS TO USA.
( ) Ø.3

[58] Suppose some factors tend to cause the ECONOMIC STATUS to be RECOVERY and some other factors tend to cause NO CHANGE, what would be the prevailing state of ECONOMIC STATUS ?

(1) RECOVERY
(2) NO CHANGE
(3) None of the above - I don't know

Enter a number

( ) 3

[59] Would you list the types of development that will contribute to the occurrence of the above possibilities

connected with the OPEC OIL PRICE.
If you don't want to explore the situations of the OPEC OIL PRICE any further, you may type $exo.
(1) <u>$inference</u>

****

Based on the data YOU provided, the likelihood of the events of the OUTCOME OF 1984 ELECTION is concluded as follows:

.727	(REAGAN RE-ELECTED)
.273	(REAGAN FAIL TO BE RE-ELECTED)

****

[59] Could you enter factors that will influence the above possibilities of the OPEC OIL PRICE.
If you don't want to explore the events of the OPEC OIL PRICE any further, you may type $exo.
(1) $end "reagan.exe"

****

The structure so far elicited has been saved on REAGAN.EXE

****

-(logout)

< Return to the Host Operating System. >

Chapter 6

CONCLUSIONS

Although CONVINCE has not been in operation for a sufficient length of time to permit exhaustive testing, it has been shown that the system provides an adequate and useful tool as a situation assessment aid. The most desirable features are its graceful user interface that allows the user to interact with the system in an English-like dialogue, and its ability to lead the dialogue with questions through an exploration of the most appropriate areas promising quick resolutions of the uncertainties connected with the major variables.

The inference scheme proposed and adopted in CONVINCE provides a means to combine synergistically causal reasoning with diagnostic reasoning. The independences assumed to facilitate evidence propagation (i.e., cross-generation, inter-cause and inter-symptom independence) are quite reasonable and compatible with human reasoning, and therefore, should lead to acceptable conclusion in many problem domains. The few test cases studied produced results that matched the user's expectations and intuitions.

6.1 CONVINCE DEFICIENCIES

Some deficiencies with CONVINCE emerged during interactions. The first deficiency results from its limitted representational structure which forces the user to model every problem using only variables and their causal relationships. Although causal relationship is the most important one in situation assessment decision-making, it alone is insufficient to achieve an expert level of performance. Additional studies are needed to find ways of integrating causal relationships with other kinds of relationships to infer more valid conclusions.

Several AI reseachers have identified the necessity for a multiple level representation of causality[Patil 81]. In this scheme, a variable is both a generic description of a number of more specific concepts and a specific description of more abstracted, generic descriptions. Although we did force the user to represent causal relationships in a uniform level of detail, this sometimes seems unnatural and too restrictive. We could extend CONVINCE's modeling power by allowing the user to represent causal relationships in multiple levels of detail. However, the use of such representations in interactive, domain independent systems requires a much larger number of queries than a single-level system does.Second deficiency in CONVINCE is its inability to check the consistency of given input data. A conditional probability matrix quantifying an incoming link to a node imposes constraints on the conditional probabilities of the other incoming links to that node. For example, consider the example of Figure 6-1 where the node ALARM SOUND (A or not A) is connected with two causal factors: BURGLARY (B or not B) and EARTHQUAKE (E or not E). Suppose that the conditional matrices P(ALARM|BURGLARY) and P(ALARM|EARTHQUAKE) are elicited independently to yield those values listed in Figure 6-1. The conditional matrix P(ALARM|BURGLARY) determines the range of the marginal probability of A, P(A):

$$(6\text{-}1) \quad 0.3 \leq P(A) \leq 0.7$$

because

$$(6\text{-}2) \quad P(A) = P(B)P(A|B) + P(\text{not } B)P(A|\text{not } B)$$

and

$$(6\text{-}3) \quad 0.0 \leq P(B) \leq 1.0 \quad .$$

At the same time, P(ALARM|EARTHQUAKE) also determines the range of P(A) such that

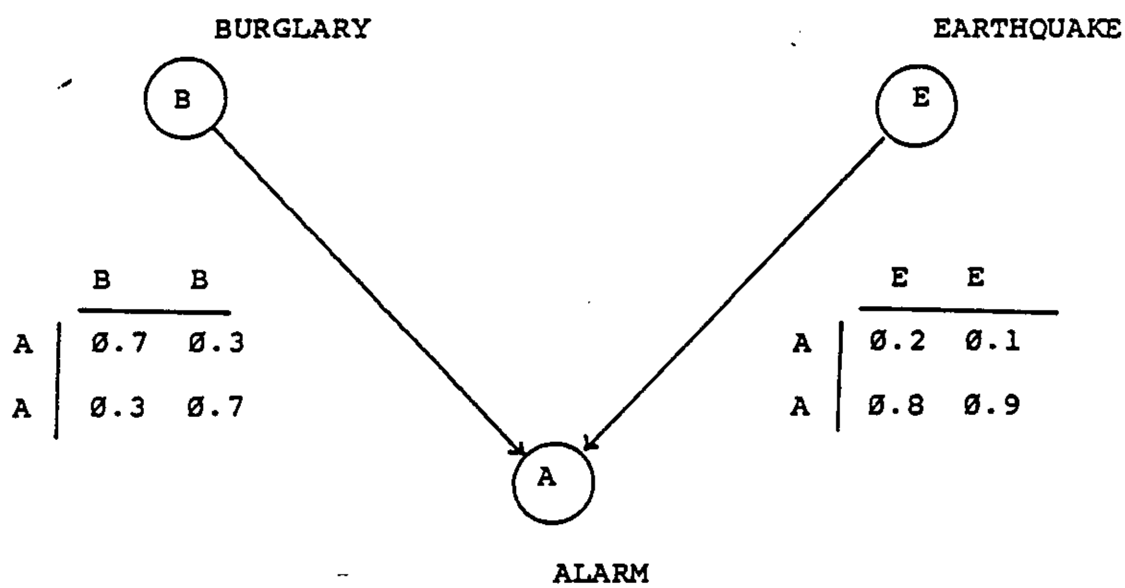

Figure 6-1 : Inconsistent Conditional Probability Matrices

(6-4) $\emptyset.1 < P(A) < \emptyset.2$.

No marginal probability exists satisfying both of Eq. (6-1) and Eq. (6-4); thus the two conditional probability matrices introduced are inconsistent with each other.

Consistency among conditional probabilities is a global property; hence inconsistencies cannot be detected by local checking. Although the validity of input data is the user's responsibility, a good decision-support system should detect inconsistencies and guide the user toward valid input.

Another issue rased in our experience with CONVINCE concerns the interpretation of the conditional probabilities given a cause. For example, when a physician is requested to estimate the likelihood of fever given a patient who has flu, he naturally assumes the question meant the likelihood of fever when the patient is known to have flu but not malaria, typhoid or pneumonia. This interpretation requires the assessment of the likelihood of an event assuming that all of the other causes are in a "standard", or "default" state, unlike the unconditional interpretation used in the causal network of CONVINCE. This kind "isolated" interpretation is often natural[1] and may resolve, if well

----

1. This kind of interpretation is indeed used in some problem domains such as failure analysis and reliability study.

formulated, the inconsistency problem we brought up previously. However, it is often hard to determine what interpretation the user perceived.

6.2 REFINING CONVINCE'S SITUATION ASSESSMENT PROCESS

A question is often raised regarding the validity of the basic assumption of decision aiding system like CONVINCE; is decomposition-and-synthesis (also called divide-and-conquer, or problem reduction) approach better than its counterpart holistic analysis ? Although some experiments may attest to the contrary[Slovic 77], the general concensus among the decision analysis and artificial intelligence communities remains that the decomposition-and-synthesis method is a useful tool for attacking complex decision problems. Based on this belief, the likelihood of a variable is computed by aggregating impacts from its neighboring variables.

However, consider the political analyst's case shown in Chapter 5. The impact of its causal factors on ECONOMIC STATUS has been estimated holistically before the two causal factors of the node are identified. This impact was than decomposed into two impacts: through OPEC OIL PRICE and through JAPANESE COOPERATION. Clearly these two causal impacts are not the only ones which make up the complex variable. Other causal factors surely exist which were not explicated probably because there are too many of them, each having only a minor significance relative to those mentioned. If we neglect those unexplicated factors, we loose information which could have been considered in the holistic judgment that was discarded. One simple solution to the problem is to create, for each variable, a hypothetical link that would summarize the impact from the unexplicated backgroung information.

The capabilities of CONVINCE can be augmented in several directions. The first might be toward analyst's

bookkeeper. This system would elicit and store not only the decision-maker's belief but also the source of his information. The source of information could be newspaper articles, intelligent reports, etc. This bookkeeping capability, when coordinated with an explanation system, would extend the reliability of the conclusion it made.

A second augmentation would be an automatic explanation generator. This would aid the user in generating structured reports whose major components would be the conclusions and the explanations about how the conclusions were derived. Information needed for an explanation would be obtained through traversing the problem network by following the causal links that connect the concluded hypothesis with the observed evidence. This traversal process could be goal-directed (i.e., from the hypothesis to the evidential observations) or data-driven (i.e., from the observed data to the conclusions), and could be performed in either a manual or automated mode. In the manual mode, the user would move a pointer throughout the network and gather information from the data attached to the network elements. He would then choose or discard information items based on his intuitive judgment of their relevancy, and compile them for textual report. In the automated mode, the system would select information items based on the strength of their evidential impact. The report could be produced in multiple levels of detail. For a full report all the paths leading to the concluded hypothesis would be included, while for a summary report only those links whose dependency is stronger than a threshold. In addition to an explanation of reasoning process, such a report could also contain auxillary information such as the source of the evidential data, its reliablity, future developments that require careful monitoring, etc.

A third extension would be an inclusion of a capability to experiment so that system could answer "what if" type questions. In the currently implemented prototype of CONVINCE, expermentations can be performed only by constructing various alternative structures through a number of interview sessions. The augmented system would allow the user to test, in a real time, on the effect of variations of a given parameter, before that parameter is fixed.

A fourth extension would be a graphics interface. Such an augmented system could display the structure so far elicited at the user's request in a graphical form during the network construction process. This feature would help the user observe the network as a whole. Problem structures could be scanned using the display as a sliding window to traverse and explore the entire network. Editing the network by graphical input could be also considered for an augmented system.

An augmented system could be designed and implemented naturally as an object-oriented system[Weireb 81, Bobrow 82, McAuther 81]. Object-oriented system design methodology is the state-of-the-art software design technique developed under AI discipline. In an object-oriented environment, system design amounts to identifying objects of the problem domain and to defining the functional behaviors of those objects. In an augmented CONVINCE system, there would be only two kinds of generic classes of objects: variables (nodes) and causal relations (links). Each object would have various slots in which information related to itself would be encoded. Behaviors would be defined for each class of objects to describe how the object would act when a message is reveived. In this manner, how an object is deleted, created or modified would be described in a unified way.

Once having a generic descriptions of object types, object instances would be created during the interview with the user. When an instance is created, it would inherit properties from their parent object. When a new variable is obtained, a node instance representing this variable would be created and messages sent to its nearest neighboring instances. Then, the instances who receive the message would update their beliefs according to the fomulae described in Chapter 3. They would send messages in turn to their neighbors. It could be imagined that each instance is associated with its own processor and possesses a decision-making capability. An instance would receive messages only from instances which are connected by a link. In this build up, the inference procedure described in Chapter 3 would be viewed as a description of objects' behavior upon receiving messages from their neighbors.

Recently, a number of object-oriented programming systems have become available on the market. An object-oriented programming system called FLAVORS is available on the Symbolic's LISP Machine[Weireb 81]. A system called LOOP has been written recently for the INTERLISP environment[Bobrow 82]. The FLAVORS system would provide an adequate programming environment for the development of an augmented CONVINCE system exploiting the LISP machine's window and graphics capability.

APPENDIX

CONSTRUCTION OF CHOW TREE

Chow and Liu [Chow 68] devised a method for optimally approximating N-dimensional discrete probability distribution by a product of second-order, pairwise distributions. The approximation is optimal in the sense that it preserves the maximum information among those distributions that can be approximated by a product of such N-1 second-order conditional distributions. They showed that the optimal approximation corresponds to the maximal spanning tree of a graph which is formed by representing variables as nodes, pair-wise relationships as links, and assigning the links by the mutual information of the two variables located at each end of the link.

In order to discuss the goodness of this approximation, Chow and Liu defined the notion of 'closeness' between two distributions based on an information theoretical measure. The closeness measure I(,) is defined as follows. Let $P(X)$ and $P_a(X)$ be two probability distributions of a discrete variable $X = (x_1, x_2, \ldots, x_n)$. The quantity:

$$(A-1) \quad I(P, P_a) = \sum P(X_i) \log [P(X_i)/P_a(X_i)]$$

has the property that:

$$(A-2) \quad I(P,P_a) \geq 0$$

with equality sign if and only if $P(X) = P_a(X)$ for all X. In other words, the closeness measure I(,) always yields a positive value except when the two distributions are identical.

The closeness measure $I(P, P_a)$ can be written as:

$$(A-3) \quad I(P,P_a) = - \sum P(X) \log P_a(X) + \sum P(X) \log P(X).$$

Since $P_a(X)$ is a valid probability distribution and a product of N-1 second order conditional distributions, we can write $P_a(X)$ as a product of N-1 second order conditional probabilities and one first order probability. Therefore:

$$(A-4) \quad P_a(X) = \prod_{i=k} P(X_k) \quad P(X_i | X_{s(i)})$$

where $P(X_i | X_{s(i)})$ is a selected second order conditional probability. If we write $P(X_k)$ as $P(X_k|X_K)$:

$$\begin{aligned}(A-5) \quad I(P,P_a) &= - \sum P(X) \sum_{i=1} \log P(X_i|X_{s(i)}) \\ &\quad + \sum P(X) \log P(X) \\ &= - \sum P(X) \sum_{i=1}^{n} \log [P(X_i, X_{s(i)}) \\ &\quad / P(X_i) P(X_{s(i)})]\end{aligned}$$

$$- \sum P(X) \sum_{i=1} \log P(X_i)$$
$$+ \sum P(X) \log P(X).$$

Recall the usual definition of mutual information. The mutual information $M(x_i, x_j)$ between two variables x_i and x_j is given by:

$$\text{(A-6)} \quad M(x_i, x_j) = \sum P(x_i, x_j) \log (x_i, x_j)/P(x_i)P(x_j).$$

The closeness measure I(,) can now be written in terms of the mutual information as:

$$\text{(A-7)} \quad I(P, P_a) = \sum_i M(x_i, x_{s(i)}) + \sum_i H(x_i) - H(x).$$

Notice that the terms $H(x)$ and $H(x_i)$'s are independent of the selection of the pairwise distributions $P(x_i, X_{s(i)})$. Thus, we can conclude from the Eq. (A-7) that maximizing the sum of the mutual information of the selected pairwise-distributions is equivalent to minimizing the closeness measure $I(P, P_a)$. Note that the mutual information measure is usually used for a measure of dependency between two variables. The measure becomes zero if the two variables are statistically independent, and the maximum when a variable is totally dependent upon the other. Minimizing the closeness measure is also equivalent to selecting the highest dependent N-1 pairs of variables.

Chow also gives an algorithm by which an unknown N-th order distribution can approximated using N-1 pairwise distributions, assuming that all pairwise distributions are available. Once the variables are represented as nodes and the pairwise distributions as links, a complete graph could be formed where each link is assigned by the mutual information of the two variables located at each end of the link. The optimal approximation corresponds to the maximun spanning tree of this graph. Kruskal's algorithm has been used for the generation of the maximum spanning tree, a tree constructed by selecting links one at a time in the order decreasing mutual information unless such a link forms a cycle.

Chow's approach is also applicable to the cases where not all pairwise distributions are avaliable. These are more typical when we deal with real world problems, such as medical diagnosis, in that only strong correlational relationships are perceived and represented. In these cases too, the link that has the highest mutual information measure is selected from the set of available links and is added to the existing network unless it forms a cycle.

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